

# NONLINEAR FINITE ELEMENT ANALYSIS OF REINFORCED CONCRETE PLATES IN PUNCHING SHEAR

Ahsanul Kabir <sup>(1)</sup> Colonel DSM Shahddeul Islam <sup>(2)</sup>

<sup>(1)</sup>Dr. Professor Department of Civil Engineering, Bangladesh University of Engineering & Technology, Dhaka, Bangladesh

<sup>(2)</sup> Senior Instructor, Department of Civil Engineering, MIST, Mirpur Cantonment Dhaka

## ABSTRACT

*This is an attempt to carry out nonlinear analysis of reinforced concrete flat plates using 3 D-Brick elements in combination with Link elements. A 3-dimensional reinforced concrete flat plate model is generated. The formulation has been checked against experimental results of tests on slabs subjected to punching shear. From a comparison of the numerical and the experimental results, it is concluded that the generated model by the Finite Element Analysis package ANSYS is capable of analysing reinforced concrete plates with significant degree of accuracy. A few parametric studies has also been undertaken to determine the behaviour of plates with and without shear reinforcements and/or compression reinforcement.*

**KEYWORDS:** Punching Shear, Nonlinear Analysis, Finite Element Analysis, Reinforced Concrete, Flat plate, 3-Dimension Compression steel.

## 1.0 INTRODUCTION

The flat plate is a very common and competitive structural system for cast-in-place slabs in buildings since no beams, column capitals, or drop panels are involved. This means that formwork becomes extremely simple leading to simplifying and accelerating site operations. Thus architecturally pleasing buildings and bridges are built at comparatively lower cost. Flat plates allow easy and flexible partitioning of space and reduce the overall height of tall buildings. However, the structural concept is at great disadvantage, because of the risk of brittle punching failure at columns.

In the design of reinforced concrete flat plates, the regions around the column always pose a critical analysis problem. Column tends to punch through the flat plates, flat slabs and footings because of the shear stresses, which act in them around the perimeter of the columns. Shear failure, both beam and punching type, may be considered more dangerous than flexure failure because of greater uncertainty in predicting shear failure, which is likely to occur suddenly with no advance warning of distress. Large research efforts have been made in the past and are still being made to develop methods for a reliable prediction of the punching shear capacity. Numerous tests have been carried out to evaluate the punching shear strength of slabs. Several theories have been put forward to predict the strength observed in these tests.

Elstner and Hognestad (1956) presented a research report on the methods and results of experimental work on the shearing strength of reinforced concrete slabs subjected to a centrally located concentrated

load. The test findings show that the shearing strength of slabs is a function of concrete strength as well as several other variables like percentage of tension reinforcement, size of column, conditions of support and amount and position of shear reinforcement. Studies have been conducted to verify the effect of punching shear resistance of flat-slab and plates with or without shear reinforcement [Harmon and Zhangyuan (1989), Yamada, et. al. (1990), Loo and Falamaki (1992), and Broms (2000)]. Several researchers investigated different methods for strengthening slab-column connections against punching (Elgabry and Ghali (1990), Olivera, et. al. (2000) and Pilakoutas and Li (2003)).

From the literature review made so far, it may be noted that two methods are available to assess the punching shear capacity of flat plates. The first is by testing of models or the prototypes. The second is numerical analysis using computer software. When both the methods have their own merits and demerits, for design and evaluation purposes, the latter one is more suitable. Yet, the core of a reliable numerical analysis lies on an adequate constitutive model that can represent the material behaviour. Further investigation is necessary in this regard and this is an effort to understand the punching behaviour of reinforced concrete flat plates using package software ANSYS.

## **2.0 OBJECTIVES**

Based on the background discussion, the present research aims at achieving the following objectives:

- a. To model numerically the punching shear behavior of reinforced concrete flat plates using finite element package ANSYS and compare various test results with finite element analysis.
- b. To determine complete load-deflection response of reinforced concrete flat plates and predict the ultimate load carrying capacity of such structures.
- c. To study the effect of flexural and shear reinforcement on the punching shear strength.

## **3.0 METHODOLOGY**

The main obstacle to finite element analysis of reinforced concrete structures is the difficulty in characterizing the material properties. Much effort has been spent in search of a realistic model to predict the behaviour of reinforced concrete structures. Due mainly to the complexity of the composite nature of the material, proper modeling of such structures is not straightforward and still an active area of research.

A nonlinear finite element programme (ANSYS) is used for this analysis. ANSYS is the most widely used general purpose software of FE analysis. It has versatile applications in the field of static and dynamic analysis of structure. ANSYS also offers linear and nonlinear (material, geometric, and boundary conditions) analysis. It provides a dedicated three-dimensional eight noded solid element, Solid 65, to

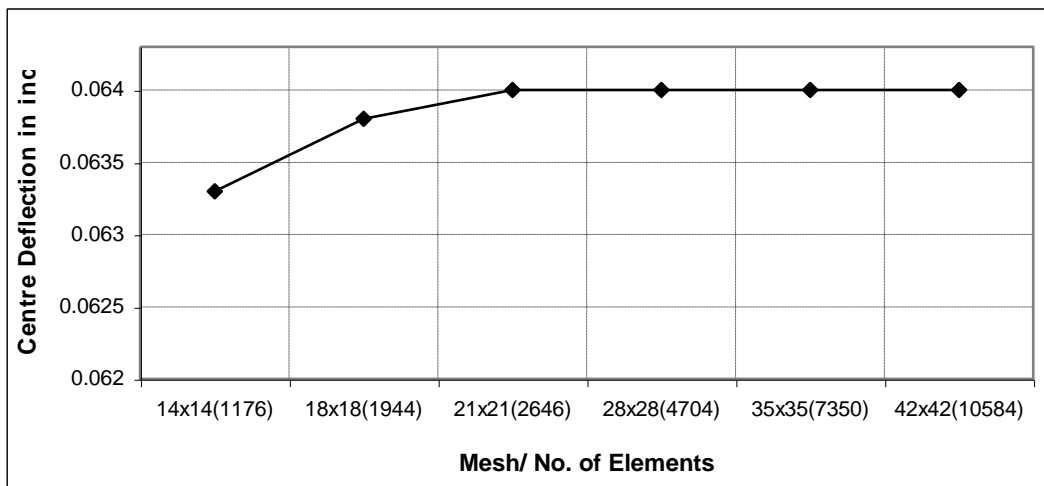
model the nonlinear response of brittle materials based on a constitutive model for the triaxial behaviour of concrete. The element includes a smeared crack analogy for cracking in tension zones and a plasticity algorithm to account for the possibility of concrete crushing in compression zones. The element behaves in a linear elastic manner until either of the specified tensile or compressive strengths are exceeded.

The steel for the finite element model is assumed as an elastic-perfectly plastic material and identical in tension and compression. The internal reinforcements are modeled using three dimensional spar elements LINK 8 with plasticity. To develop a realistic model, link elements are used as discrete reinforcement connecting solid element nodes rather than smeared reinforcement. In this study, perfect bond between materials is assumed. To provide the perfect bond, the link elements for the steel are connected between nodes of each adjacent concrete solid element, so that the two materials share the same nodes. The nonlinear effects due to the cracking and crushing of concrete and the yielding of steel reinforcement is included. The model is a smeared crack model, in the sense that it would not track individual macro cracks. An incremental finite element technique has been used which simulates the nonlinear load-deflection behaviour of reinforced concrete structure.

The effects of some numerical parameters are studied in order to establish the stability of the overall solution process and as a basic guide for subsequent analytical problems. The numerical parameters selected for sensitivity analysis of the solution procedure are; load increment size, element mesh size, shear transfer coefficient, and convergence criteria.

#### **4. FINITE ELEMENT DISCRETISATION**

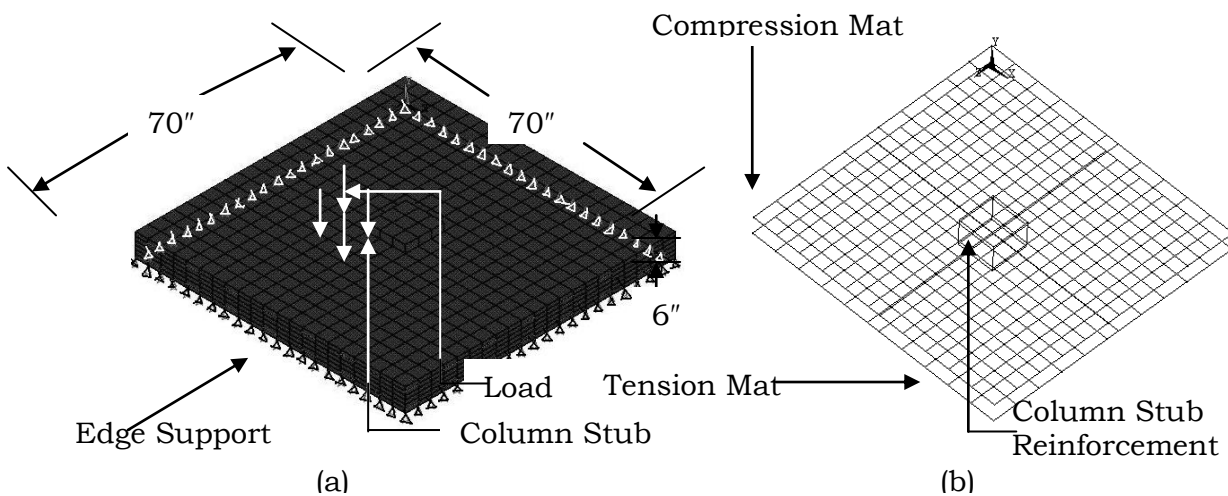
As an initial step, a finite element analysis requires meshing of the model. In other words, the slab model is divided into a number of small elements. A convergence of results is obtained when an adequate number of elements are used in a model. This is practically achieved when an increase in the mesh density has a negligible effect on the results. Optimum meshing was checked by elastic analysis for constant load from the plot of centre deflection verses number of mesh as shown in Figure 4.1. The figure shows that the difference in deflection is negligible when mesh density in plain increased from 21x21 to 28x28. The 21x21 mesh model with six layers across the thickness is equivalent to 2646 concrete elements selected for the model slabs.



**Figure 4.1 Mesh Optimisation**

### 5.1 FINITE ELEMENT MODEL GEOMETRY

To study the effect of varying support condition, full-size plate with dimensions of 70" x 70" x 6" is used. The plate is supported at the edges and loaded through a centrally located column stub of 10" x 10" as indicated in Figure 5.1 (a). Typical reinforcement for tension and compression mat is shown in Figure 5.1(b)



**Figure 5.1 (a) Model of the Plate (b) Typical Tension and Compression Mat.**

### 5.2 NONLINEAR MATERIAL MODELLING STRATEGIES AND FEM INPUT DATA

The requirement to include the 3 D nonlinear response of reinforced concrete in capturing the ultimate response demands the use of the dedicated Solid 65 element in ANSYS. The internal reinforcements were modelled using three dimensional spar elements with plasticity, the Link 8 embedded within the solid mesh. This option was favoured over the alternative smeared stiffness capability as it allowed the reinforcement to be precisely located whilst maintaining a relatively coarse mesh for the surrounding concrete medium. The inherent assumption is that there is full displacement compatibility between the reinforcement and the concrete and that no bond slippage occurs.

ANSYS require input data for concrete material properties as: Elastic modulus ( $E_c$ ), ultimate uniaxial compressive strength ( $f'_c$ ), ultimate tensile strength (modulus of rupture,  $f_r$ ), Poisson's ratio ( $\nu$ ), shear transfer coefficient ( $\beta_t$ ) and compressive uniaxial stress-strain relationship for concrete. A summary of the concrete properties used in this study is shown in Table 5.2.1. For this purpose, four slabs from Elstner and Hognestad (1956) were taken as reference. Value of ultimate uniaxial compressive strength ( $f'_c$ ) is obtained from the test data of Elstner and Hognestad (1956). Values of  $E_c$  and  $f_r$  are computed using ACI 318-99 Code Equations as  $E_c = 57500\sqrt{f'_c}$  and  $f_r = 7.5\sqrt{f'_c}$ .

**Table 5.2.1: Concrete Material Properties Used**

| Plate No. | $E_c$ (ksi) | $f'_c$ (psi) | $f_r$ (psi) | $\nu$ | $\beta_t$ |
|-----------|-------------|--------------|-------------|-------|-----------|
| A-1a      | 2600        | 2040         | 339         | 0.17  | 0.35      |
| A-7b      | 3660        | 4050         | 477         | 0.17  | 0.35      |
| B-14      | 4920        | 7330         | 642         | 0.17  | 0.35      |
| B-16      | 4920        | 7330         | 642         | 0.17  | 0.35      |

In a nonlinear reinforced concrete analysis, the shear transfer coefficient must be assumed. The shear transfer coefficient,  $\beta_t$ , represents conditions of crack face. The value of  $\beta_t$  ranges from 0.0 to 1.0, with 0.0 representing a smooth crack (complete loss of shear transfer) and 1.0 representing a rough crack (no loss of shear) (ANSYS 1997). In this study, based on a number of trials a value of 0.35 was used for  $\beta_t$ , which resulted in quite good predictions. Values less than 0.3 were tried, but they caused divergence problems at very low loading levels.

To construct the uniaxial compressive stress-strain curve for concrete in this study, numerical expressions Equation 5.2.1 and 5.2.2 [Desayi and Krishan (1964)], were used along with Equation 5.2.3 [Gere and Timoshenko(1997)].

$$f = \frac{E_c \varepsilon}{1 + (\varepsilon / \varepsilon_0)^2} \quad (5.2.1)$$

$$\varepsilon_0 = \frac{2f'_c}{E_c} \quad (5.2.2)$$

$$E_c = \frac{f}{\varepsilon} \quad (5.2.3)$$

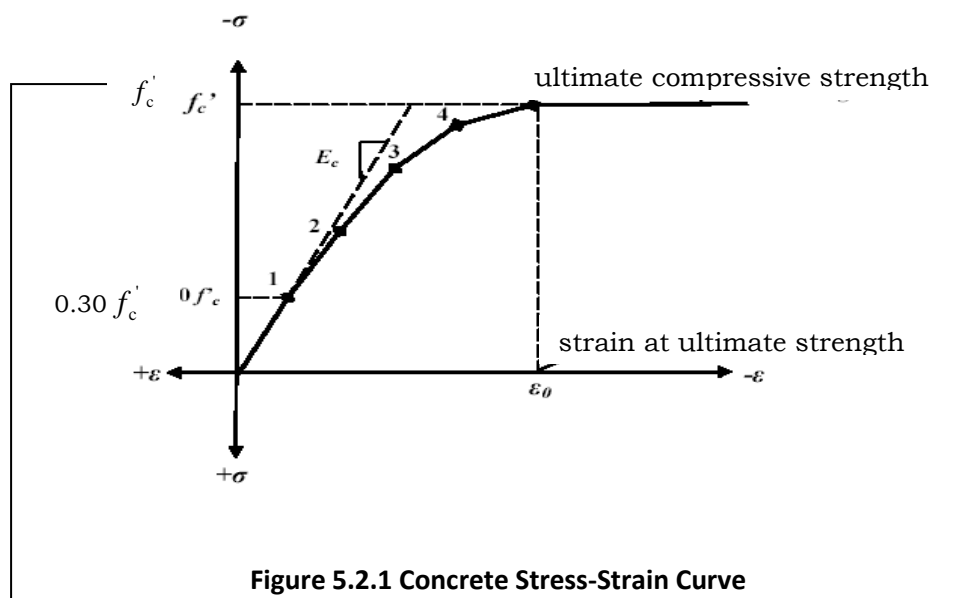
Where:

$f$  = stress at any strain  $\mathcal{E}$ , psi

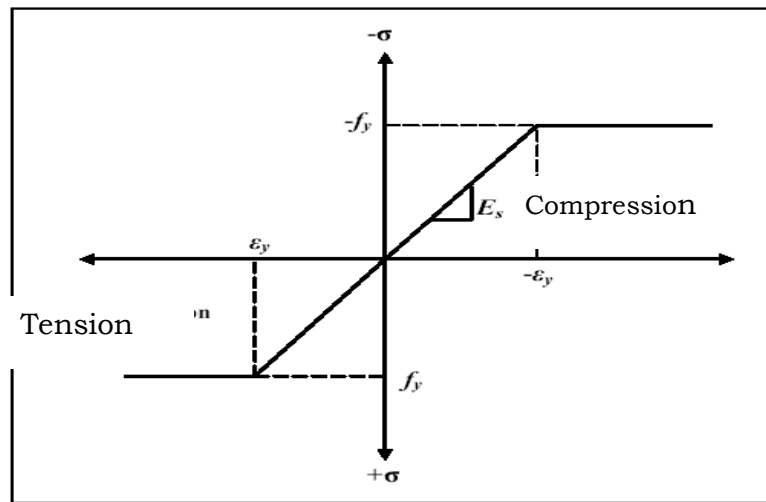
$\mathcal{E}$  = strain at stress  $f$

$\varepsilon_o$  = strain at the ultimate compressive strength  $f'_c$

Figure 5.2.1 shows the typical compressive uniaxial stress-strain relationship that was used in this study. The curve starts at zero stress and strain. Point no. 1, at  $0.30 f'_c$ , is calculated for the stress-strain relationship of the concrete in the linear range (Equation 5.2.3). The last point is at  $\varepsilon_o$  and  $f'_c$ . The remaining points are obtained from Equation 5.2.1, in which  $\varepsilon_o$  is calculated from Equation 5.2.2. An assumption was made of perfectly plastic behaviour after the last point.



Elastic modulus and yield stress, for the steel reinforcement used in this study follow the corresponding material properties used for the experimental investigation by Elstner and Hognestad (1956). The steel for the numerical analysis is assumed to be an elastic-perfectly plastic material and identical in tension and compression. Poisson's ratio of 0.3 was used for steel. Figure 5.2.2 shows typical stress-strain relationship for steel reinforcement.



**Figure 5.2.2. Stress-Strain Curve for Reinforcement**

### 5.3 NONLINEAR SOLUTION STRATEGIES

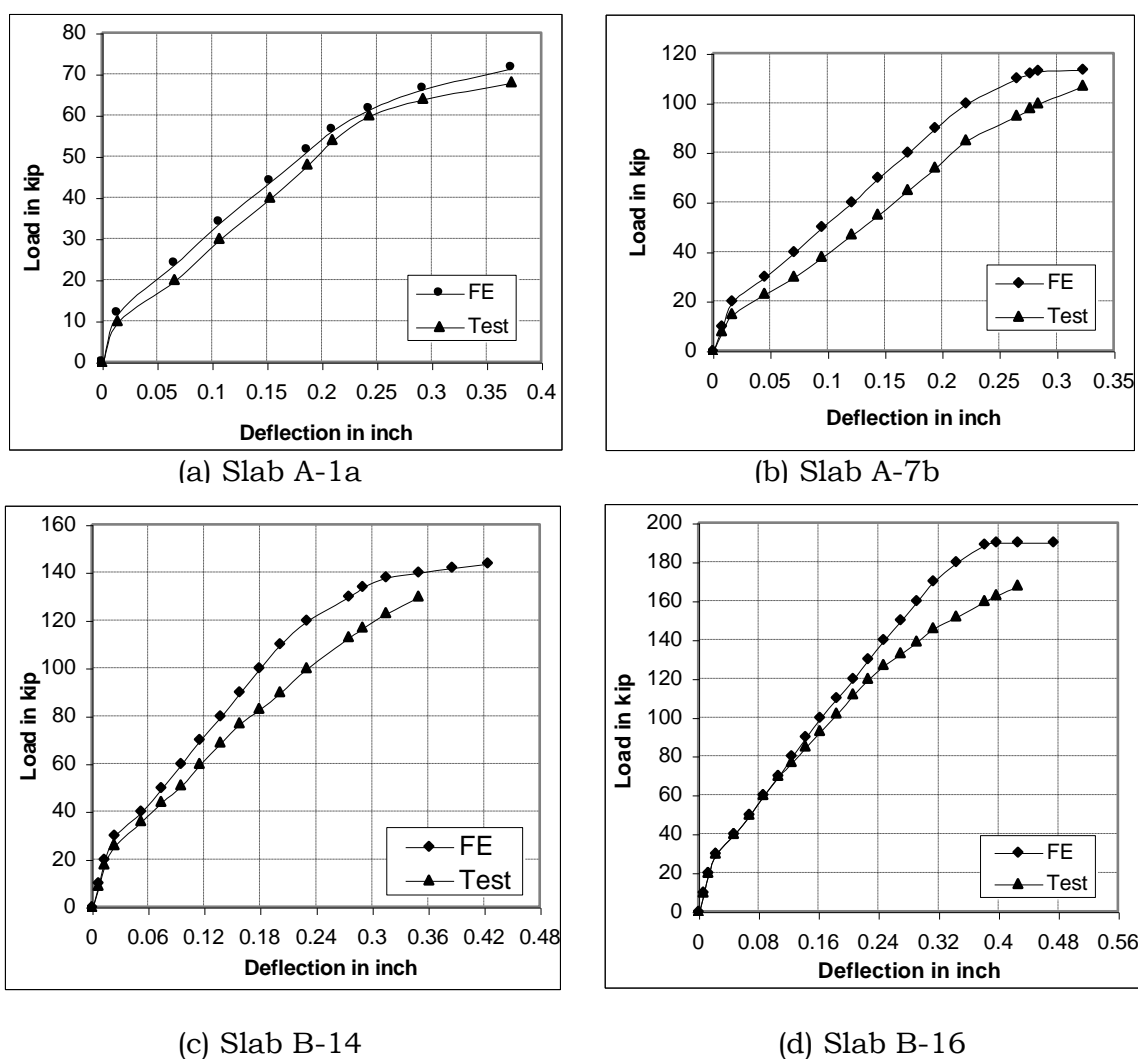
Nonlinear solution technique and overall nonlinear solution strategy to be adopted are very important for nonlinear pre and post-yielding analyses of concrete members. In nonlinear solution, the total load applied to a model is divided into a series of load increments called load steps. At the completion of each incremental solution, the stiffness matrix of the model is adjusted to reflect nonlinear changes in structural stiffness before proceeding to the next increment. The ANSYS programme uses Newton-Raphson equilibrium iterations for updating the model stiffness. Newton-Raphson equilibrium iterations provide convergence at the end of each load increment within prescribed tolerance limit. In this study, Full Newton-Raphson option with Sparse Direct Solver was used for speed and robustness of the solution.

The decision about the convergence criterion to be used in the nonlinear iterative solution strategy is also important. Choice of convergence criterion depends on the type of structure, degree of accuracy required, efficiency in solution process required etc. ANSYS offers various options of convergence criterion. For the reinforced concrete solid elements, convergence criteria were based on force and displacement, and convergence tolerance limits were initially selected by the ANSYS. It was observed that convergence of the solutions was difficult to achieve and took long computation time. Using only displacement checking with convergence tolerance limit increased to maximum of 5 times the default (=1%) tolerance limits, computation time reduced to almost  $1/3^{\text{rd}}$  with nearly similar results. Therefore, convergence criteria are based on displacement with tolerance limit 0.05 in order to save time.

For the nonlinear analysis, automatic time stepping in the ANSYS programme predicts and controls load step sizes. The maximum and minimum load step sizes are required for automatic time stepping. Considering the behaviour of reinforced concrete element under load, large maximum number of substeps are chosen to apply the load in small enough increments to ensure that analysis closely follow the actual structures load-response curve.

## 6.0 COMPARISON OF EXPERIMENTAL AND NUMERICAL RESULTS

To carry out finite element analysis in order to predict the behaviour of any structure, it is essential to verify the developed model against some theoretical or test results to ensure that the developed model is tracing the actual response closely. Results from the nonlinear finite element analysis are compared with the test results obtained from Elstner and Hognestad (1956) to ensure the acceptability of the obtained results. Comparative load-deflection response for the test and FE results is shown in Figure 6(a) through 6(d) for A-1a, A-7b, B-14 and B-16 slabs of Elstner and Hognestad (1956) respectively. These four test slabs cover a wide range of concrete strength, flexural reinforcement ratio, strength of reinforcement and effect of shear reinforcement.



**Figure 6. Comparison of Experimental and FE-analysis Load-Deflection Curves**

It is observed that Non-linear load-deflection curves show very close results at every stages of load history of the slabs up to failure. The FE analysis can trace the test results closely. The initially linear relation experience a small jump with a sudden loss of stiffness, when cracking in concrete begins followed by a nearly linear response with decreased slope. The almost bilinear response of the finite element model is consistent with the test data. Hence it may be concluded that the FE-program can be



used to simulate the whole load-deformation curve which include the elastic part, the initiation of cracking, shear cracks, crushing, and the yielding of the steel bars.

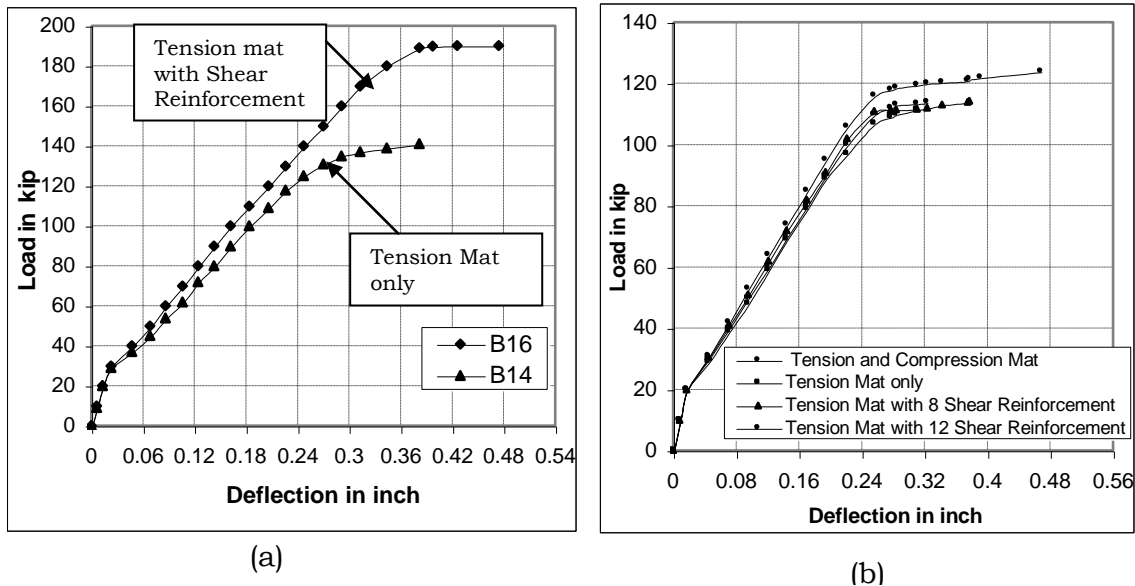
## **7.0 DISCUSSIONS ON RESULTS**

The general behavior of the finite element models represented by the load-deflection plots at centre of plate show good agreement with the test data. However, the finite element models show slightly stiffer response than the test data in both the linear and nonlinear ranges. The effects of bond slip (between the concrete and steel) and microcracks occurring in the actual plates are not included in the finite element models. These have some positive contribution to the higher stiffness of the finite element models. The final loads from the finite element analyses are higher than the ultimate loads from the experimental results by 5% to 15%.

Clearly, the correlation of test and numerical data depends on the assignment of accurate linear and nonlinear material properties as appropriate. In general, given the compressive strength of the concrete, it is thus usually possible to arrive at a sensible set of material data for inclusion in the nonlinear numerical model. Convergence of the solution is usually achieved with an adequate number of elements to map the test model. For reinforced concrete models in a nonlinear analysis, however, too fine of a mesh may cause numerical instability. On the other hand, if the mesh is too coarse, it may not yield good result. In this study, the minimum FE element size used is 3.33" x 3.33" x 1".

For nonlinear analysis of a reinforced concrete plate, the total load applied to a model must be divided into a number of load steps. Properly defining minimum and maximum sizes for each load step, assists in better convergence of the solutions and reduces computational time.

Parametric study is conducted to observe the effect of compression mat and shear reinforcement. Effect of shear reinforcement is observed by plotting the load-deflection curve of plate B-14 and B-16 together as shown in Figure 7 (a). The punching shear strength and the ductility of the slab increased with the addition of shear reinforcement. Next a parametric study is conducted to observe the effect of compression mat and shear reinforcement on plate A-7b. First group of 8 bars was bend directly under the column edge and the second group of additional 4 bars was bend at 3.333 inch from the column edge. From Figure 7(b), it can be seen that the effect of compression mat on the ultimate load is less significant, better effect can be attained by shear reinforcement. It is also observed from the load-deflection response of Figure 7(b) that bending the bars directly under column edge has insignificant effect on the strength enhancement compared to bending those at certain distance away from the column edge.



**Figure 7** Effect of Shear and Flexural Reinforcement.

## 8. CONCLUSIONS

From the comparison of the numerical results with experimental data following conclusions may be drawn:

- i. The good agreement obtained between the numerical and the experimental results establish the validity and acceptability of the computational model.
- ii. The finite element models show slightly stiffer response than the test data in both the linear and nonlinear ranges.
- iii. Load-deflection curves show very close results at the early stages of load history for all the analyses conducted. The initially linear relation experiences a small jump, with a sudden loss of stiffness, when cracking in concrete begins, followed by a nearly linear curve with flatter slope than the initial.
- iv. The influence of flexural reinforcement on compression face is not so significant on the ultimate load.
- v. The addition of shear reinforcement has a significant effect on the punching shear strength and ductility of the plates. Bending the bent-bar shear reinforcement at certain distance from the column edge has better effect than bending directly under column edges.

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