MODIFIED EXTREMUM DIFFERENCE METHOD (MEDM) FOR SOLVING COST MINIMIZING TRANSPORTATION PROBLEMS

Md. Amirul Islam(1), Md. Munir Hossain(1), Md. Alamgir Hussain(1)

1Department of Mathematics, Military Institute of Science and Technology. Mirpur, Dhaka, Bangladesh.

Email: amirulmaths01@yahoo.com,ripon.raeef@gmail.com, alamgir3766@yahoo.com

ABSTRACT

Transporting commodities from a number of sources to a number of destinations with minimum Transportation Cost is known as Transportation Problem. Minimization of Transportation Cost is the main object of Transportation Problem and Optimal Solution is searched in order to do this. Initial Basic Feasible Solution is a must for finding the Optimal Solution for Transportation Problem. In this article Modified Extremum Difference Method is proposed to obtain a better Initial Basic Feasible Solution for Transportation Problem. A comparative study is also carried out to justify the performance of the proposed method.

Mathematics Subject Classification: 90B50, 90C08.

Key Words: Transportation Problem, Transportation Cost, Initial Basic Feasible Solution, Optimal Solution.

1.0 INTRODUCTION

Special type of linear programming problem is known as Transportation Problem (TP). Transportation Problems basically discuss the distribution plan of a product from a number of sources (e.g. factories) to a number of destinations (e.g. warehouse). Main objective of TP is to minimize transportation cost.

F.L. Hitchcock [1] in 1941 presented a study entitled 'The Distribution of a Product from Several Sources to Numerous Localities' which is known as the first formal contribution to the TP. In 1947, T.C. Koopmans [2] presented a study called 'Optimum Utilization of the Transportation System'. Systematic procedure for finding solution for TPs were developed, primarily by Dantzig [3] in 1951, and then by Charnes, Cooper and Henderson [4] in 1953.

TP is also studied since long [6-14], but the well-known existing methods for finding an Initial Basic Feasible Solution (IBFS) for the TPs are North West Corner Method (NWCM) [5], Least Cost Method (LCM) [5], Vogel's Approximation Method (VAM) [5] and Extremum Difference Method (EDM) [5]. Among these methods, usually VAM and EDM yield better Initial Basic Feasible Solution.

Recently, Ahmed. M.M. et al [14] proposed a computationally easier solution procedure for Transportation Problems which is known as 'Incessant Allocation Method (IAM)'. In this paper Modified Extremum Difference Method (MEDM) is proposed to obtain a better Initial Basic Feasible Solution which is computationally easier. A comparative study is also carried out to justify the method and it is found that the performance of proposed method is better.

2.0 CONCEPT OF TRANSPORTATION PROBLEM

The Transportation Problem is concerned with finding an optimal distribution plan for a single commodity such that total transportation cost is minimum or the time taken in the shipment is minimum. However, Transportation problem is one of the subclasses of Linear Programming Problems in which the objective is to transport various quantities of a single homogeneous commodity that are initially stored at various origins to different destinations in such a way that the total transportation cost is minimum. To achieve this objective we must know the amount and location of available supplies and the quantities demanded. In addition, we must know the costs those results from transporting one unit of commodity from various origins to various destinations. Few examples of transportation problem are summarized in Table 2.1.

Source	Destination	Commodity	Objective
Plants	Markets	Finished goods	Minimizing total cost of shipping
Plants	Finished goods warehouses	Finished goods	Minimizing total cost of shipping
Finished goods warehouses	Markets	Finished goods	Minimizing total cost of shipping
Suppliers	Plants	Raw materials	Minimizing total cost of shipping
Suppliers	Raw material's warehouses	Raw materials	Minimizing total cost of shipping
Raw materials warehouses	Plants	Raw materials	Minimizing total cost of shipping

Table 2.1

A given supply of the commodity is considered that is available at a number of sources, there is a specified demand for the commodity at each of a number of destinations, and the Transportation Cost between each source-destination pair is known. In the simplest case, the unit transportation cost is constant. The problem is to find the optimal distribution plan for transporting the products from sources to destinations that minimizes the total Transportation Cost.

3.0 NETWORK REPRESENTATION OF TRANSPORTATION MODEL

We can generally represent the transportation model by the network in Figure-3.1.1. There are m sources and n destinations, each represented by a node. The arcs represent the routes linking the sources and destinations. Arc (i,j) joining source i to destination j carries two pieces of information: the Transportation Cost per unit, c_{ij}, and the amount shipped, x_{ij}. The amount of supply at source i is a_i, and the amount of demand at destination j is b_j. The objective of the model is to determine the unknowns x_{ij} that will minimize the total Transportation Cost while satisfying the supply and demand restrictions.

Transportation Network: The general TP is represented by the network as below:

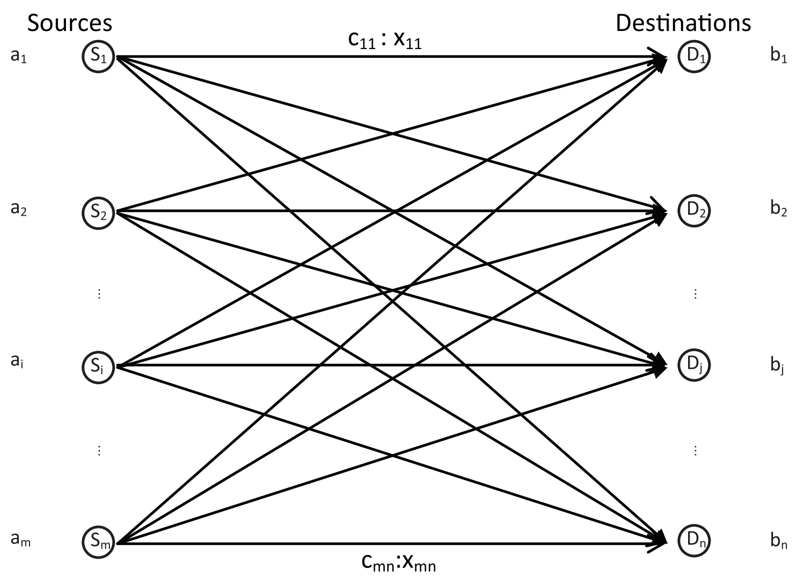


Fig: 3.1-1

4.0 TABULAR FORM OF TRANSPORTATION PROBLEM

The tabular form of a Transportation Problem is matrices within a matrix. Cost matrix is one of them that representing unit Transportation Cost c_{ij}, indicating the cost of shipping a unit from the i-th origin to the j-th destination. Super impose of this matrix is the matrix of transportation variable x_{ij}, indicating the amount shipped from i-th source to j-th destination. Right and bottom sides of the transportation table point out the amounts of supplies ai available at source i and the amount demanded b_j at destination j.

6.0 COST MINIMIZATION TRANSPORTATION PROBLEM

Classical Transportation Problem is a particular class of Linear Programming Problem, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of m origins (e.g., factories) to a set of n destinations (e.g., shops) to meet the specific requirements.

	1	2	...	...	n	Supply
1	x_{11}	x_{12}	...	...	x_{1n}	a_1
	c_{11}	c_{12}	...	...	c_{1n}	
2	x_{21}	x_{22}	...	...	x_{2n}	a_2
	c_{21}	c_{22}	...	...	c_{2n}	
3	...	...	...	...	...	a_3
	...	...	...	...	...	
.	...	...	...	...	...	.
.	...	...	...	...	...	.
.	x_{m1}	x_{m2}	...	...	x_{mn}	.
m	c_{m1}	c_{m2}	...	...	c_{mn}	a_m
Demand	b_1	b_2	...	...	b_n	

5.0 MATHEMATICAL FORMULATION OF TRANSPORTATION PROBLEM

The Transportation Problem can be stated as an allocation problem in which there are m sources (suppliers) and n destinations (customers). Each of the m sources can allocate to any of the n destinations at a per unit carrying cost c_{ij} (unit transportation cost from source i to destination j). Each sources has a supply of a_i units, and each destination has a demand of b_j units, . The objective is to determine which routes are to be opened and the size of the shipment on those routes, so that the total Transportation Cost of meeting demand, given the supply constraints, is minimized.

In other words, Transportation Problems deal with the transportation of a single product manufactured at different plants (supply origins) to a number of different warehouses (demand destinations). The objective is to satisfy the demand at destinations from the supply constraints at the possible minimum Transportation Cost. To achieve this objective, we must know the quantity of available supplies and the quantity demanded. A cost minimization Transportation Problem is formulated as:

Minimize:

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le a_i \quad ; \quad i = 1, 2, \dots \dots, m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \quad ; \quad j = 1, 2, \dots \dots, n$$

$x_{ij} \ge 0$ for all i and j.

Where i =1, 2,…,m is the set of origins.

j =1, 2,…,n is the set of destinations.

x_{ij} = the quantity transported from the *i*-th origin to the *j*-th destination.

c_{ij} = per unit cost in transporting goods from i-th origin to the *j*-th destination.

a_i = the amount available at the *i*-th origin.

b_j = the demand of the *j*-th destination.

7.0 BASIC STEPS OF SOLVING TRANSPORTATION PROBLEM

The basic steps for obtaining an optimum solution to a Transportation Problem are:

Step-1: Mathematical formulation of the Transportation Problem.

- Step-2: Verify the model of the Transportation Problem either it is balanced or unbalanced. If the problem is unbalanced first make it balanced.
- Step-3: Determine the starting/initial basic feasible solution. The transportation model has (m + n -1) independent equations, which means that the starting basic solution consists of (m + n -1) basic variables. In a mxn Transportation Problem the solution is said to be feasible if (m + n - 1) cells are occupied.
- Step-4: Verify the optimality condition of the starting/initial feasible solution. If the solution is not optimal improve it for obtaining optimal solution.

8.0 PROPOSED METHOD

Various steps of the proposed method are summarized below.

- **Step-1:** Formulate the problem mathematically and balance the problem if unbalance.
- **Step-2:** Determination of first allocation:

  Determine the penalty cost for each of the rows by taking the difference between the highest and lowest cell cost along the same row, and put it on the right of the corresponding rows of the cost matrix. These numbers are called Row Penalties (RP). In the similar fashion, calculate the Column Penalties(CP) for each of the columns and write them below the corresponding columns of the cost matrix.
- Choose the highest penalty cost and observe the row or column along which it appears. If a tie occurs, choose any one of them randomly.
- Allocate maximum to the cell having lowest unit transportation cost in the row or column along which the highest penalty cost appears.

Step-3: Determination of rest of the allocations:

- Adjust the supply and demand requirements in the respective rows and columns. Then following cases arise :
- Case-1: If the allocation $x_{ij}=a_i$, i-th row is to be crossed out and bj is reduced to (b_j - a_i). Now complete the allocation along *j*-th column by making the allocation/allocations in the smallest cost cell/cells continuously. Consider that, j-th column is exhausted for the allocation x_{kj} at the cell (k, j). Now, follow the same procedure to complete the allocation along k-th row and continue this process until entire rows and columns are exhausted. Again if the allocation x_{ij} = b_j, just reverse the process for x_{ij} = a_i .
- Case-2: If the allocation x_{ij} = a_i -b_j, find the next smallest cost cell, (i , k) from the rest of the cost cells along i-th row and j-th column. Assign a zero in the cell (i, k) and cross out i-th row and j-th column. After that complete the allocation along

k-th column following the process described in Case-1to complete the allocations.

Case-3: For any allocation, made along the row/column satisfies both the row and column, in such case find the smallest cost cell which is along the column/row and assign a zero in that cell and continue the process described in case-1 to case-3 to complete the allocations

Step-4: Finally calculate the total transportation cost.

9.0 NUMERICAL EXAMPLES

9.1 Example-1: A company manufactures motor tyres and it has three factories F_1, F_2 and F_3 whose weekly production capacities are 150, 175 and 275 thousand pieces of tyres respectively. The company supplies tyres to its three showrooms located at D_1, D_2 and D_3 whose weekly demands are 200, 100 and 300 thousand pieces respectively. The transportation cost per thousand pieces of tyre is given below in the Transportation Table (TT):

		Showrooms			
		D_1	D_2	D_3	Supply
Factories	F_1	6	8	10	150
	F_2	7	11	11	175
	F_3	4	5	12	275
Demand		200	100	300	

We want to schedule the shifting of tyres from factories to showrooms with a minimum cost.

9.2 Example-2: Consider the following data for Transportation Problem for the company stated above:

		Showrooms				
		D_1	D_2	D_3	D_4	Supply
Factories	F_1	4	6	8	8	40
	F_2	6	8	6	7	60
	F_3	5	7	6	8	50
Demand		20	30	50	50	

9.3 Example-3: Consider the following transportation problem:

		Showrooms				
		D_1	D_2	D_3	D_4	Supply
Factories	F_1	7	5	9	11	30
	F_2	4	3	8	6	25
	F_3	3	8	10	5	20
	F_4	2	6	7	3	15
Demand		30	30	20	10	

9.4 Solution of Example 9.1 is shown below:

Since the factories demand 600 units equals the total supply 600 units, the given problem is a balanced TP.

The distribution made according to proposed algorithm is

	Destinations				
Sources	**1**	**2**	**3**	**Supply**	**RP**
1	6	25 / 8	125 / 10	**150**	4
2	7	11	175 / 11	**175**	4
3	200 / 4	75 / 5	12	**275**	8
Demand	**200**	**100**	**300**		
CP	3	6	2		

The total transportation cost is,

25x8+125x10+175x11+200x4+75x5=4550.

10.0 COMPARISON OF TRANSPORTATION COST OBTAINED BY DIFFERENT METHODS

A comparative study among the solutions obtained by proposed method and the other existing methods is also carried out for the examples 9.1, 9.2 and 9.3 that are shown in the tables 10.1, 10.2 and 10.3

Table: 10.1

Method	Example 9.1
NWCM	5925
LCM	4550
VAM	5125
EDM	4550
MEDM	4550
Optimal Solution	4525

Fig : 10.1

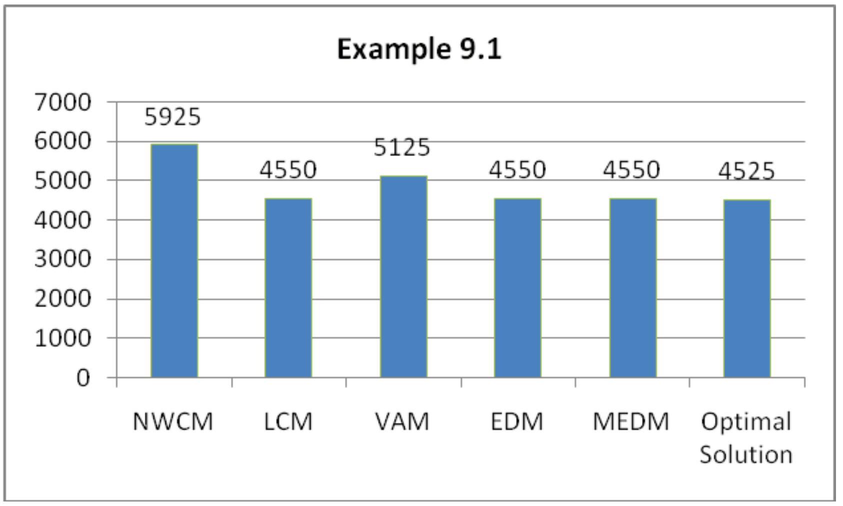


Table : 10.2

Method	Example 9.2
NWCM	980
LCM	960
VAM	960
EDM	960
MEDM	920
Optimal Solution	920

Fig : 10.2

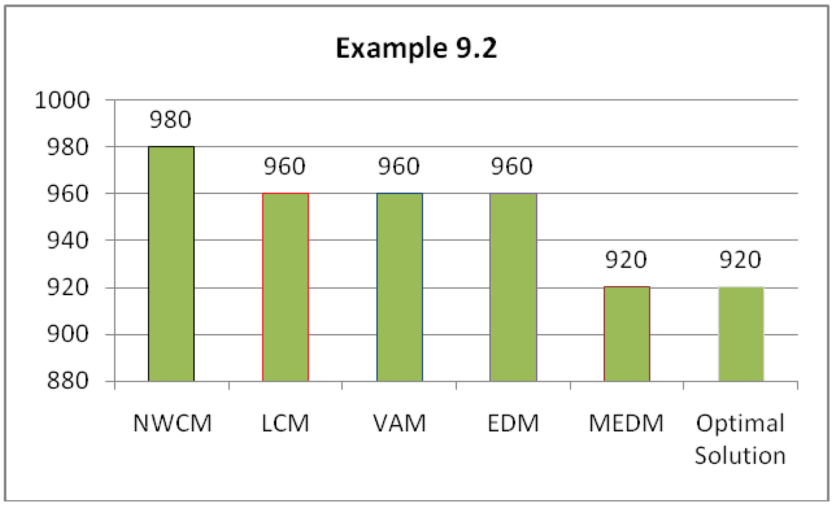
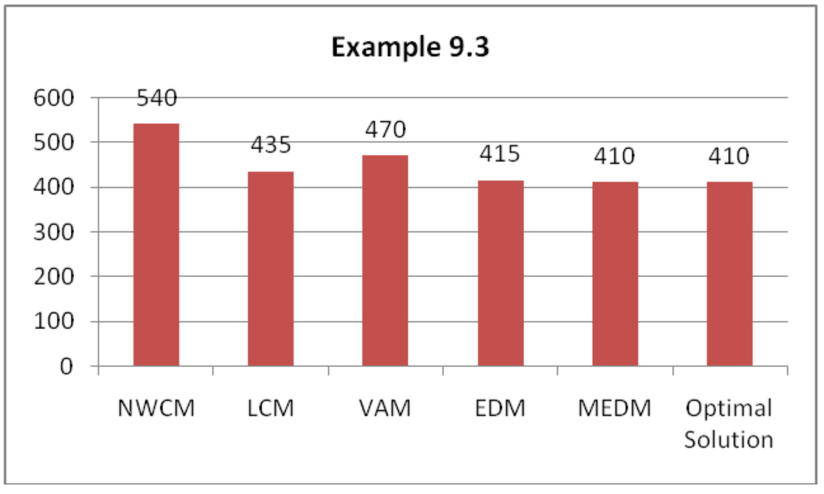


Table : 10.3

Method	Example 9.3
NWCM	540
LCM	435
VAM	470
EDM	415
MEDM	410
Optimal Solution	410

Fig : 10.3



From the tables 10.1, 10.2 and 10.3 it is clear that the performance of the MEDM is better or equal of the existing methods including EDM. This performance is also shown in bar-chart to get the clear idea about the performance of proposed method.

11.0 CONCLUSION

Transportation model provides a powerful framework to select the supply route with minimum Transportation Cost of an industry. The proposed method can be one of the solution procedures to select this route.

Proposed method of finding IBFS for the minimization of Transportation Cost is illustrated numerically. It is observed that proposed algorithm provides comparatively better IBFS than those obtained by the traditional algorithms which is either optimal or near to optimal solution. Basically the proposed method is a modified version of EDM. It is also observed that the proposed method performs either same or better than the EDM, but the computation procedure is easier than the existing EDM as the penalty is required to find only for the first allocation.

Finally, it may be conclude that our proposed algorithm provides a remarkable IBFS by ensuring minimum Transportation Cost which may be an attractive alternative to the traditional methods in solving transportation problems.

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