APPLICATION OF MATHEMATICS IN ENGINEERING; A SPECIFIC EXAMPLE IN UNDERWATER ROBOTICS

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ABSTRACT

Mathematics is the base of all engineering. To become an engineer, one must have sound knowledge in Mathematics. However, at the begging of engineering study, most of the students have misconception about Mathematics. They consider it as non-departmental subject and provide less importance of learning it. They may have general conception that mathematics is important in engineering; however they don't know the specific application in their particular engineering subjects. In the long run they face difficulties to solve engineering problems which need a lot of mathematics. The purpose of the paper is to relate different topics of mathematics to different engineering applications. In this paper, the main concentration is given to the application of basic topics of mathematics in some common engineering fields. In addition, the application of mathematics in developing a squid-like underwater robot has been discussed in detail for better understanding. It is hoped that the engineering students will be motivated to understand the importance of mathematics in their engineering study. It is also expected that mathematics lecturers would be encouraged to teach mathematical problems related to the particular engineering fields.

Key Words: Mathematics, Engineering, Application, Underwater robotics.

1.0 INTRODUCTION:

Let us start the discussion with two questions: First, who is the father of Mathematics? An obvious question, but difficult to answer. Imagine the ancient time, humans were just like other mammals – Hunt-Eat-Sleep; but still somehow we created numbers to satisfy the creative needs of human brain. It all started with early man, who by, some means wanted to have some record of their members in their group, amount of food shared, how many children do they have, etc. Then they developed the fundamental operations of mathematics; addition, subtraction, multiplication and division. Men grew and they required more mathematical calculations, simple to complex and finally an era of modern technology came.

The second question: what is engineering? According to the definition given by the Accreditation Board for Engineering and Technology (ABET), *"Engineering is the profession in which a knowledge of the mathematical and natural sciences is applied with judgment to develop ways to utilize, economically, the materials and forces of nature for the benefit of mankind."* Simply it can be defined as, *"Engineering is the application of mathematics and science to create something from the natural resources"*. From both of the definitions it is clear that mathematics becomes essential for engineering; it is impossible to engineer something without the help of mathematics. Engineers depend on mathematics as they are engaged in research, development and production; to do this, they have to solve various complex problems. Indeed design and automation of a complex system requires a systematic, global and formal approach which required mathematics.

Mathematics is the brain of all engineering; no engineering study is possible without mathematics. However, at the beginning, most of the engineering students couldn't realize its importance in their study. There are very few papers published by focusing the importance of mathematics in engineering; some examples are listed [1,6]. Mathematics is a wide discipline and it has many forms. Different forms of mathematics may be applied to achieve different solutions of the problems in various engineering fields. It is not possible to summarize all of them in one article. In this paper, the discussion will be concentrated to the application of basic topics of mathematics to some common engineering fields. Linear algebra, geometry, calculus, differential equation, vector analysis, complex analysis, numerical analysis are usually taught in engineering disciplines as they are important to understand the engineering subjects such as fluid mechanics, heat transfer, electric circuits, mechanics of materials, structures etc. On the other hand, civil, mechanical, electrical and electronics are known as the core engineering subjects. Computer engineering is treated as general engineering as it is applied in all engineering. Besides these, there are some multidisciplinary engineering subjects developed by the combination of basic engineering to deal with specific problems; such as, Naval Architecture and Ocean Engineering (NAME), Aeronautical Engineering (AE), Biomedical Engineering (BME), Nuclear Science Engineering (NSE), Petroleum and Mining Engineering (PME), Industrial Production Engineering (IPE), Environmental Engineering etc. In this paper, the discussion will be limited to the application of the basic topics of mathematics to the common engineering fields. Finally, we'll discuss the application mathematics in developing underwater squid-like robot as a particular example.

2.0 BRANCHES OF MATHEMATICS:

Mathematics is classified into two prime branches: Pure Mathematics and Applied Mathematics; the second one is mostly important in engineering. The first two formal branches of mathematics were Algebra and Geometry. Geometry is one of the most important and exciting ideas of mathematics. It provides a connection between algebra and geometry through graphs of lines and curves. This enables geometric problems to be solved algebraically and provides geometric insights into algebra. The invention of Calculus was an extremely important development in mathematics that enabled mathematicians and physicists to model the real world in ways that was previously impossible. The word calculus comes from Latin meaning *"small stone"*, because it is like understanding something by looking at small pieces. Calculus has two sub-branches: Differential Calculus and Integral Calculus. Differential calculus cuts something into small pieces to find how it changes where as integral calculus joins small pieces together to find how much there is. Other two significant branches of applied mathematics are Ordinary Differential Equation (ODE) and Partial Differential Equation (PDE), which are frequently used in engineering. ODE and PDE are formed and also solved by using calculus. The mathematical computation is common term in most of the engineering; the Computational Fluid Dynamics (CFD) is nothing but deal with some of some PDE's. Beside these, there are some branches of applied mathematics which are regularly used in different engineering, e.g., Trigonometry, Vector Analysis, Complex Variable, Matrix, Laplace transformation, Fourier analysis etc.

3.0 APPLICATION OF MATHEMATICS IN VARIOUS ENGINEERING FIELDS:

3.1 Civil Engineering: Civil engineers are responsible to design and build small to mega structures, such as buildings, bridges, dams, highways, tunnels etc. To do this, civil engineers use nearly every form of mathematics. Algebra and Geometry are used on a daily basis; calculus, differential equations and trigonometry are also used frequently. Geometry is used to design structures and ensure that it perform in a safe and meaningful way. For the shapes used, a civil engineer must understand and know how to compute such quantities as lengths, areas, volumes, centroid, moments of inertia, and must be able to determine the spatial relationship among these shapes. For this purpose, they have to depend on calculus. The civil engineers also depend greatly on the trigonometric and geometric logics. They use trigonometry often when surveying a structure. Surveying deals with land elevations as well as the various angles of structures. When a bridge is being designed, physics is used to figure out how large the supporting piers should be, as well as how thick the steel columns of bridge need to be, and how many of them should be installed. Physics equations typically use algebra, calculus, and trigonometry. When designing a bridge, civil engineers may use differential equations to calculate the approximate size of the supporting piers needed. The civil engineers also use complex mathematical modeling to predict the life span of the structure to be constructed.

3.2 Mechanical Engineering: Mechanical engineering is one of the broadest engineering disciplines. The Mechanical Engineers work on power-producing machines such as electric generators, internal combustion engines, and steam and gas turbines. They also work on power-using machines such as refrigeration and air-conditioning equipment, machine tools, material handling systems, elevators and escalators, industrial production equipment, and robots used in manufacturing. Mechanical engineers research, design, develop, manufacture, and test tools, engines, gears, machines, and other mechanical devices. To do these works they need most of the branches of mathematics. In mechanical engineering, mathematics is important because it is required to solve problems, to analyze mathematical relations and in using the laws of nature, which are mathematical expressions. Mechanical engineers require algebra and trigonometry frequently. They use geometry in theory of mechanics, machine design, drafting. Calculus is used almost all areas of Mechanical engineering for theoretical part and very much used in Fluid Mechanics and Thermodynamics. The knowledge of differential equation is required in Heat Transfer, Fluid Dynamics, CFD, Mechanical vibrations etc.

3.3 Electrical and Electronic Engineering: Electrical engineers design, develop, test, and supervise the electrical equipment. Some of this equipment includes electric motors and machinery controls, lighting and wiring in buildings, automobiles and aircrafts, navigation systems. Power generation, control, and transmission devices are also supported by electric utilities. The main study in electrical and electronic engineering should be begun after one has mastered the basic concepts of physics, which require mathematics from the arithmetic to algebra and trigonometry, through differential and integral calculus and differential equations. Electrical engineering entails a lot of critical analysis, crypto analysis, operation research and management; for this purpose it heavily utilizes the mathematical principles, logic, formulae, and calculations. If one is interested in electronics only as a hobby then general math may be okay. However, if you are serious about becoming an electronic and electrical engineer then you must be good at mathematics. Electrical engineers need advanced mathematics training through calculus; because, basic electronics involves the use of it. Ohms law requires a basic knowledge of algebra to fully understand it and to be able to use it effectively. Some knowledge of trigonometry would be helpful. Electrical engineers need to know how to calculate various rates of change in electrical parameters in a quick and relatively simple manner. Without the appropriate skills of mathematics an electrical engineer will be greatly handicapped in his work.

3.4 Computer Science and Engineering: Mathematics is one of the foundations of computer sciences and engineering. Computer sciences heavily rely on algorithms, which the latter in turn heavily relies on mathematics. Many of the functions and operators in all programing languages require some knowledge in mathematics. A beginner in programming may not need mathematics, but as the programmer advances through the level of difficulty, he / she will have to use more advanced mathematics. 'Theoretical computer science' strongly involves discrete mathematics. Discrete mathematics is basically the study of mathematical structures that are discrete rather than continuous, and so this 'theoretical' branch of computer sciences involves a lot of mathematics, in the form of graphs, algorithms, computational geometry, quantum computation, algebra, computational number theory. Boolean algebra is the basis of digital logic design, a key component of building computers. Many applications such as calculators, video games and graphical applications are compelled to the use of mathematics. In chip technology, microcomputers and software engineering need more complex algorithm feasible in many different areas like measurement technique, information processing, signal processing, automation, telecommunication, data protecting, and robotics. It is said, without proper knowledge of mathematics, any computer scientist would be on the path to failure. Hence, mathematics provides the right pathway towards success in computing. So to conclude, mathematics is considered a necessity for any successful computer scientist.

3.5 Interdisciplinary Engineering: Besides the basic engineering fields discussed above, there are some advanced engineering subjects deal with particular problems. These engineering subjects are known as interdisciplinary engineering as it is required the knowledge of many basic engineering disciplines. The basic engineering subjects cannot be thought without Mathematics; so its importance is obvious to the interdisciplinary engineering also. The applications of mathematics to some of those engineering subjects are discussed as below:

(Aeronautical Engineering) is one of the interdisciplinary engineering subjects concerned with the development of aircraft and spacecraft. Mathematics is a key foundational component of all engineering, and aeronautical engineering is no exception. Aircraft stability and dynamics are not possible to understand without mathematics. Whether modeling shapes, designing on a computer, checking stresses and strains, calculating fluid dynamics or determining areas, mathematics is the root of all these activities. Flow problems are solved with Computational Fluid Dynamics, which is nothing but the solutions of Navier-Stokes equations. Trigonometric functions are used to estimate distances and landing patterns and navigate around obstacles. There is no aerodynamics without the fluid mechanics and mathematical descriptions of Prandtl, and von Karman.

(Biomedical Engineering or Bio-Engineering) is the application of engineering principles to the fields of biology and health care. Mathematics is the fundamental tool of biomedical engineering. When analyzing and designing medical solutions, biomedical engineers routinely use calculus and geometry skills. , For instance, in designing an artificial heart, the engineer must use advanced mathematics to consider the fit of the organ in the body and its rate of pumping blood.

(Nuclear Engineering) is the study of the atomic world. Mathematics is required for all nuclear engineering, but some branches use Mathematics more than others. Mathematics is specifically used in the computer coding of nuclear engineering, the instrumentation and control (I&C) areas, and the heat transfer and fluid flow areas. Neutron transport, shielding work, nuclear criticality, and nuclear safety also involve a lot of mathematics. Algebra, Geometry, Trigonometry, Calculus, Differential equations, Statistics are mostly used in nuclear engineering study.

(Petroleum and Mining engineering) is another prominent engineering engaged in research and development of petroleum and other natural resources. Improvements in mathematical computer modeling, materials and the application of statistics, probability analysis, and new technologies like horizontal drilling and enhanced oil recovery, have drastically improved the toolbox of the petroleum engineer in recent decades.

(Production engineering) is possible only through advanced mathematics concepts. Production engineers use matrix algebra techniques for practical manufacturing applications. They need knowledge of infinite series and their convergence in order to understand common limitations that arise manufacturing-based mathematical modeling. For example, they may study tests of convergence, such as integral tests and comparison ratios, and they may learn about absolute and conditional convergence. Production engineers learn the concepts and consequences of improper integrals and error functions in engineering applications. When it comes to variables, they use partial derivatives, homogeneous functions, implicit functions and methods of undetermined multipliers.

4.0 APPLICATION OF MATHEMATICS IN UNDERWATER ROBOTICS:

In previous sections, application of mathematics in different engineering fields has been discussed. In this section, the application of mathematics to underwater robotics will be discussed. Underwater robotics belong to the Naval Architecture and Ocean Engineering. The Squid Robot, a flat fish-like underwater robot with two undulating side fins similar to that of a Stingray or a Cuttlefish has been under active investigation for many years in Osaka University, Japan (Fig-1). The author did research on the robot and achieved masters and PhD degree on it. The discussion of this section will be based on his 5 years working experience.

Fig 1: The Squid Robot of Osaka University, Japan

The work of robotics can be classified into three types: Construction of mechanical body, development of control system and mathematical computation or simulation. The knowledge of mechanical engineering is required to build the body of the robot by joining different parts. The precise mathematical calculations are required to build and connect different parts and maintain their consistency. The knowledge of geometry, trigonometry and calculus is required to develop the optimum body shape. Another challenge of robotic research is to develop an efficient control system for smooth operation, which is related to Electrical and Electronics Engineering (EEE). The importance of mathematics in electrical and electronics engineering has been discussed in the previous section. From this discussion it is easily understood that mathematical knowledge is obvious for developing effective control system. Another important sector of underwater robotics is mathematical computation and simulation which are mainly based on mathematics. Mathematical computation is a cost effective and time saving analysis to develop an optimum body of the robot.

Beside the development of mechanical body and control system, the mathematical model of the Squid robot has also been developed for more precise analysis. The experimental and numerical studies are complementary to one another. Experiments can give the total force characteristics but for the detail analysis of force the numerical study is required. The computational analysis helped to develop the optimum design of the robot, whereas the motion simulation ensured the efficient maneuverability.

The numerical computation was conducted on the Squid robot with side fins for different aspect ratios, fin angle and different frequencies [7]. The objective of this study was to design the optimum body shape and efficient fin shape for thrust generation. The flow field and hydrodynamic forces around the body of the robot have been computed by the Computational Fluid Dynamics (CFD) code to understand the flow physics around undulating side fins and to make the simple relationship between the fin's principal particulars and hydrodynamic forces. The CFD analysis is nothing but solving the Navier Stokes (N-S) equation which is belongs to Partial Differential Equation (PDE). The N-S equations and continuity equation for incompressible and unsteady flow considered in that study were represented by (Eq. 1~4):

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\frac{1}{R_n}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \quad (1)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{R_n}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \quad (2)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{R_n}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \quad (3)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (4)$$

The Navier-Stokes equations and continuity equation were solved numerically. The Finite Analytic Method for space discretization and Euler Implicit Scheme for time discretization along with the PISO algorithm for velocity pressure coupling were used in the computation. For the computation around a moving body, the equations were transformed to moving general curvilinear coordinate. The equations were transformed from the physical domain in Cartesian coordinates (x, y, z, t) into the computational domain in non-orthogonal curvilinear coordinates (ξ, η, ζ, τ). A partial transformation was used in which only the independent variables were transformed, leaving the velocity components u_i in Cartesian coordinates. Using the transformed relations the momentum (Eq. 1~4) equations were written in general form:

$$g^{11}\phi_{\xi\xi}+g^{22}\phi_{\eta\eta}+g^{33}\phi_{\zeta\zeta}=A\phi_{\xi}+B\phi_{\eta}+C\phi_{\zeta}+D\phi_{\tau}+S_{\phi} \quad (5)$$

The time history of fin angle in x-direction ϑ (x,t) was expressed as follows:

$$\theta = \theta_m \sin(2\pi nt - 2\pi x) \quad (6)$$

$$\theta_m = \arcsin\left[\left\{1 - 0.905\left(x - 0.5\right)^2\right\}\sin\Theta\right] * I \quad (7)$$

$$I = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{t}{T_{ini}}\right)\pi;\ \text{where}, 0 \leq t \leq T_{ini};\ t \geq T_{ini}, I = 1 \quad (8)$$

The hydrodynamic force produced by the undulating side fins was computed to examine the relation between the thrust coefficient (*Kx*) and advance coefficient *(J)*, similarly as in propeller chart. The thrust force *(Tx)* was estimated by subtracting the flat plate resistance from the total x-directional force in the similar manner as the experiment. In the calculation an average value was taken from a conversed period. The thrust coefficient *Kx* in *x*-direction and advanced coefficient were computed as follows (Eq. 9-10).

$$K_x = \frac{T}{\rho n^2 L^4} = \frac{T}{\rho U^2 L^2} \times \frac{U^2}{n^2 L^2} = T_x \times J^2 \quad (9)$$

$$J = \frac{1}{n}\left(= \frac{U}{NL}\right) \quad (10)$$

The coefficients are discussed in detail if the previous study of the author's group. [8]. It is seen that the Navier Stokes equations are nothing but Partial Differential Equation, which is based on differential calculus. For the solution of these equations, the knowledge is Integral Calculus, Algebra, Matrix, Determinant are required. The thrust force generated by undulating side fins has been calculated by multiplying the pressure difference between upper and lower surface of the fin with the normal vector at every point. The computed pressure distribution on the Squid robot is shown in Fig-2. The fin open characteristic (Thrust coefficient vs Advance coefficient) of the undulating fin propulsion system of the Squid robot is shown in Fig-3.

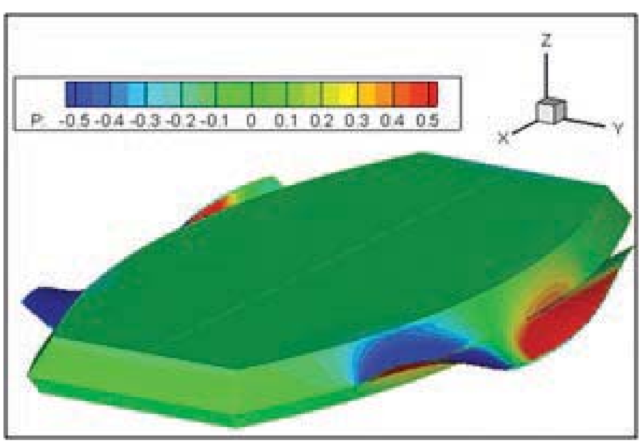

Fig 2: Pressure Distribution of the Squid Robot.

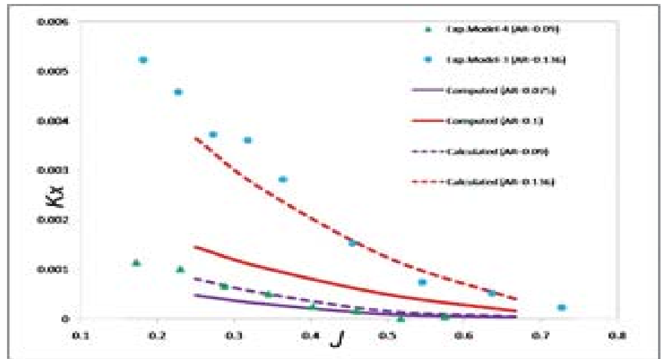

Fig 3: Fin Open Characteristic of undulating fin propulsion system.

In motion simulation, first the breaking performance of the Squid Robot has been tested [9]. The braking performance is very important performance as well as the cruising speed because the short term and short distance stopping after discovering the target is very important in the disturbed flow for exploration. The simulation was carried out for the translational motion and rotational motion. The results were found by solving the equations of motion and the hydrodynamic coefficients were obtained based on quasi-steady assumption from the towing tank captive test and the Computational Fluid Dynamics (CFD) computations of fluid force around the side fins. The space fixed coordinate system (X,Y,Z) and body fixed coordinate system (x,y,z) were used in the simulation. For the following equations (Eqs. 11-13 for translation and 14-16 for rotational motion) were considered:

$$\frac{dx}{dt} = u \quad (11)$$

$$(M + M_x)\frac{du}{dt} = F_x \quad (12)$$

$$F_x = \left(f_{xpr} + f_{xpl}\right) - C_x . u . |u| \quad (13)$$

$$\frac{d\psi}{dt} = r \quad (14)$$

$$(I_{zz} + J_{zz})\frac{dr}{dt} = T_z \quad (15)$$

$$T_z = \left(f_{xpr} - f_{xpl}\right) . \left(\frac{bB}{2} + \frac{fB}{2}\right) - C_\psi . r . |r| \quad (16)$$

The notations of the terms were detail discussed in the paper [9]. It is clearly understood the equations considered in the simulation of breaking performance and its solution were fully based on mathematical calculation.

In the second simulation study, the 6-DOF mathematical model of the Squid robot in 3D space was investigated with the aim of developing a real time simulator [10]. The real time handling simulator was developed based on the mathematical model by using Open Dynamic Engine (ODE). The 6-DOF equations of motion were solved using the mathematical model. The Hydrodynamic coefficients were obtained from the towing tank captive test and the numerical computation of fluid flow around the fins under the quasi steady assumption. The relationship between space fixed coordinate system and body fixed coordinate system can be expressed as follows (Eq. 17 - Eq.22):

$$\frac{dX}{dt} = u \cdot \cos\theta\cos\psi + v \cdot (\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi) + w \cdot (\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi) \quad (17)$$

$$\frac{dY}{dt} = u \cdot \cos\theta\sin\psi + v \cdot (\sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi) + w \cdot (\cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi) \quad (18)$$

$$\frac{dZ}{dt} = -u \cdot \sin\theta + v \cdot \sin\varphi\cos\theta + w \cdot \cos\varphi\cos\theta \quad (19)$$

$$\frac{d\varphi}{dt} = p + q \cdot \sin\varphi\tan\theta + r \cdot \cos\varphi\tan\theta \quad (20)$$

$$\frac{d\theta}{dt} = q \cdot \cos\varphi - r \cdot \sin\varphi \quad (21)$$

$$\frac{d\psi}{dt} = q \cdot \sin\varphi/\cos\theta + r \cdot \cos\varphi/\cos\theta \quad (22)$$

The first three equations represent the translation in x, y and z directions respectively and the latter three represent the gyration around x, y and z axes respectively. By solving these equations (Eq. 17 -Eq. 22), the new gravity center position in space fixed coordinate system and Eulerian angles were found. The screen image of the real time simulator are shown in Fig 4.

Fig 4. Picture of the real time simulator of Squid Robot.

5.0 CONCLUSION

It is known that there is no engineering without mathematics; to become a good engineer one must have comprehensive knowledge on mathematical. However, most of the time the students cannot correlate the mathematics courses to their engineering study and they give less importance in it. In this paper, application of different topics of mathematics to various engineering field has been discussed. Both mathematics and engineering are vast and interconnected discipline; it is quite difficult to cover all in one article. The discussion was limited to the main topics of mathematics and the common engineering fields. Application of mathematics to some advanced and interdisciplinary subjects are also pointed out. Finally, an example of direct use of mathematics to the mathematical computation and motion simulation of an underwater robot has been discussed. This paper would be a good source of information for the engineering students, specially the beginners. It is expected that the students would realize the necessity of mathematics and they could correlate their mathematical knowledge to their particular engineering study.

References

[1] M. Affandi, S. N. Aqmariah, S. Khalid and N. Affandi, Applications of Mathematics in Various Engineering Fields, ICOMEIA 2014, At Pulau Pinang, Malaysia, 2014.

[2] Vandewalle, Trends of the Need of Mathematics for Engineering and the Impact on Engineering Education, Katholieke Universiti Leuven, Kardinaal Marcierlaan 94, Belgium, 1994.

[3] Barun, M., Differential Equations and their Applications, Springer Science + Business Media, New Work 1993.

[4] Xin-She Yang, Engineering Mathematics with Examples and Applications, e-book (1st edition), 2017.

[5] Technical Note, Application of Mathematics in Engineering and Science, School of Engineering and Science, Aalborg University, Denmark, 2012.

[6] Horwitz, A., and Ebrahimpour, A., Engineering Application of Differential and Integral Calculus, Int J. of Engng, Vol 18, No. 1, Pp 78-88, 2002.

[7] Md. Mahbubar Rahman, Yasuyuki Toda, Hiroshi Miki, Computational Study on A Squid-Like Underwater Robot with Two Undulating Side Fins. Journal of Bionic Engineering, Vol. 8, pp. 25-31, 2011.

[8] Md. Mahbubar Rahman,Yasuyuki Toda, Hiroshi Miki, Study on the Performance of the Undulating Side Fins with Various Aspect Ratio using Computed Flow, Pressure Field and Hydrodynamics Forces, The 5th Asia-Pacific Workshop on Marine Hydrodynamics. Sakai, Japan, 2010.

[9] Md. Mahbubar Rahman,SinpeiSugimori, Hiroshi Miki, Risa Yamamoto, Yugo Sanada and Yasuyuki Toda, Braking Performance of the Undulating Fin Propulsion System of a Biomimetic Squid-Like Underwater Robot, Journal of Bionic Engineering Vol. 10, No.3 pp. 265-273, 2013.

[10] Md. Mahbubar Rahman, Hiroshi Miki, Shinpei Sugimori, Yugo Sanada and Yasuyuki Toda, Development of a Real Time Simulator Based on the Analysis of 6-Degrees of Freedom Motion of a Biomimetic Robot with Two Undulating Side Fins, Journal of Aero Aqua Bio-mechanisms. Vol. 3, No. 1, pp 71-78, 2013.