STUDY THE IMPACT OF HEAT GENERATION AND VARIABLE VISCOUS EFFECT ON STRETCHING SHEET IN PRESENCE OF MAGNETIC FIELD WITH VARIABLE THERMAL CONDUCTIVITY

Mohammad Shah Alam (1), Mohammad Ali (2), Md Abdul Alim (3)

1-2 Dept of Mathematics, Chittagong University of Engineering and Technology (CUET), Chittagong, Bangladesh.
E-mail: 2ali.mehidi93@gmail.com

3 Dept of Mathematics, Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh.

ABSTRACT

The aim of the study is to investigate the effect of variable viscosity and thermal conductivity on unsteady MHD boundary layer flow of an incompressible, electrically conducting and viscous fluid over a stretching sheet with heat generation by Quasi-linearization technique.The governing partial differential equations are transformed into ordinary differential equations by using similarity transformation and stretching variable. The governing momentum boundary layer and thermal boundary layer equations with the boundary conditions are transformed into a system of first order ordinary differential equations which are then solved numerically by using Runge-Kutta fourth-fifth order method along with shooting technique. The effects of the flow parameters on the velocity and temperature are computed, discussed and have been graphically represented in figures for various values of different parameters. The results presented graphically illustrate that velocity field decrease due to increasing of Magnetic parameter, unsteadiness parameter and heat source parameter and reverse trend arises for the increasing values of variable viscosity parameter and variable thermal conductivity parameter. The temperature field decreases for the effect of variable viscosity parameter and variable thermal conductivity parameter but the temperature field increases for the increasing values of magnetic parameter, unsteadiness parameter, heat source parameter, whereas for the effect of Prandtl number the temperature profile is increased on the surface of the plate and decreased away from the plate.

Key Words: MHD; stretching sheet; viscosity; Quasi-linearization technique; thermal conductivity.

1.0 INTRODUCTION

The problems of heat transfer are based on the constant physical properties of the free stream fluid. These properties may be changed with temperature, especially for fluid viscosity. In this regard, the variation of viscosity is necessary for the flow and heat transfer rates. In the application point of view, the flow over a stretching sheet is an important problem in many industrial sector such as extrusion, paper production, the hot rolling, wire drawing, glass fiber production, manufacture of plastic and rubbersheets, cooling of a large metallic plate in a bath, which may be an electrolyte, etc. located at a finite distance away. The MHD boundary layer flow of heat and mass transfer problems about an stretching sheet have become in view of its significant applications in industrial manufacturing processes such as plasma studies, petroleum industries, Magneto-hydrodynamics power generator, cooling of Nuclear reactors, boundary layer control in aerodynamics. The MHD flow in electrically conducting fluid can control the rate of cooling and the desired quality of product can be achieved. In this regard many investigators have studied the boundary layer flow of electrically conducting fluid, heat and mass transfer due to stretching sheet in presence of magnetic field. The similarity solution for unsteady momentum and heat transfer flow whose motion is caused solely by the linear stretching of a horizontal stretching surface have studied by Elbashbeshy and Bazid [1] , MHD visco-elastic fluid flow through a porous medium over a stretching sheet with chemical reaction is studied by Saleh et al [2], Ali et al. [3] analyzed the Radiation and thermal diffusion effects on a steady MHD free convection heat and mass transfer flow past an inclined stretching sheet with Hall current and heat generation, Ibrahim and Shanker [4] investigated the effect of heat source or sink on unsteady MHD boundary layer flow and heat transfer over a stretching sheet by Quasi-linearization technique, Ishak [5] studied unsteady laminar MHD flow and heat transfer due to continuously stretching plate immersed in an electrically conducting fluid, Ebashbeshy and Aldawody [6] analyzed the unsteady boundary layer flow over a stretching surface with variable heat flux in presence of heat source or sink. In presence of induced magnetic field, the steady MHD boundary layer flow and heat transfer of a viscous and electrically conducting fluid over a stretching sheet studied by Fadzilah et al. [7], Modather et al. [8] studied the effect of chemical reaction on temperature dependent viscosity and thermal conductivity on the unsteady flow and heat transfer of a micropolar fluid over a stretching sheet. Pantokratoras [9] studied the effect of variable viscosity on mixed convection heat transfer along a vertical moving surface. The present work is focused on the impact of heat generation and variable viscous effect on stretching sheet in presence of magnetic field with variable thermal conductivity.

2.0 MATHEMATICAL MODEL OF THE PRESENT PROBLEM

Consider a two dimensional unsteady laminar MHD viscous incompressible electrically conducting fluid and variable thermal conductivity on a non-conducting stretching sheet with heat generation, X- direction is taken along the leading edge of the inclined stretching sheet and Y is normal to it and extends parallel to X-axis. A magnetic field of strength B_0 is introduced to the normal to the direction to the flow. The uniform plate temperature T_w ($>T_\infty$), where T_∞ is the temperature of the fluid far away from the plate. Let u and v be the velocity components along the x and y axis respectively in the boundary layer region. Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, and energy under the influence of externally imposed magnetic field with variable viscosity and thermal conductivity in the boundary layer are:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho} \qquad (2)$$

Energy Equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty) \qquad (3)$$

Using free stream $u=U(x,t)=\dfrac{bx}{1-\gamma t}$, in equation (2), we get

$$\frac{\partial u}{\partial t}-\frac{\partial U}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$
$$-\frac{\sigma B_0^2}{\rho}(u-U) \qquad (4)$$

where u and v are the velocity components along x and y directions, T, T_w and T_∞ are the fluid temperature, the stretching sheet temperature and the free stream temperature respectively while κ is the thermal conductivity of the fluid, c_p specific heat with constant pressure, γ is the constant, μ is the coefficient of viscosity, ν is the kinematic viscosity, σ is the electrical conductivity, ρ is the fluid density, B_0 is the magnetic field intensity, U is the free steam velocity respectively. The above equations are subject to the following boundary conditions:

$$u=u_w(x,t), v=0, T(x,t)=T_w(x,t) \text{ at } y=0$$
$$u=U(x,t), T=T_\infty(x,t) \text{ at } y\to\infty \qquad (5)$$

The velocity of the sheet $u_w(x,t)$, the surface temperature of the sheet $T_w(x,t)$, and the transverse magnetic field strength $B(t)$ are respectively defined as follows :

$$u_w=\frac{ax}{1-\gamma t}, T_w-T_\infty=\frac{bx}{1-\gamma t}, B(t)=\frac{B_0}{1-\gamma t},$$

where, a and b are constants, $a>0, b\geq 0$, both a and γ have dimension $(\text{time})^{-1}$. The flow is caused by the stretching of the sheet which moves in its own plane with velocity

$u_w = \frac{ax}{1-\gamma t}$, where stretching rate 'a' and γ are positive constant. Following Arunachalam [10] and Chaim [11], the thermal conductivity k is assumed to vary as a linear function of temperature and taken of the form $k = k^*(1+ \theta)$ and assume that the fluid properties are isotropic and constant except for the fluid viscosity μ which is assumed to vary as a linear function of temperature and taken as the form $\mu = \mu^*(1+a^*\theta)$. Where κ and μ are the variable thermal conductivity and variable viscosity. Also k^* and μ^* are the thermal conductivity and coefficient of viscosity and a* and ε are the fluid characteristic measure of the steepness of relation between the viscosity and thermal conductivity with temperature. We introduce the steam function $\psi(x,y)$ as defined by $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$.

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$\psi = x\sqrt{\frac{a\nu}{1-\gamma t}}\, f(\eta), \eta = \sqrt{\frac{a}{\nu(1-\gamma t)}}\, y,\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained

$$(1+a^*\theta)f''' - f'' - f'^2 - A\left(f' + \frac{1}{2}\eta f''\right) -$$

$$M(f' - \lambda) + A\lambda + \lambda^2 = 0 \text{ ---------------------------- } (6)$$

$$(1+\varepsilon\theta)\theta'' - (A\eta + 1)\Pr f\,\theta' + Q^*\theta = 0 \text{ ---------- } (7)$$

The transform boundary conditions:

$$f = 0\ f' = 1, \theta = 1 \text{ at } \eta = 0,$$

$$f' = \lambda, \theta = 0 \text{ as } \eta \to \infty \text{ ---------------------------- } (8)$$

Where f' and θ are the dimensionless velocity and temperature respectively, η is the similarity variable, the prime denotes differentiation with respect to η. Also

$$M = \frac{\sigma B_0^2(1-\gamma t)}{\rho a}, A = \frac{\gamma}{a}, \lambda = \frac{b}{a}, \Pr = \frac{\mu c_p}{\kappa^*},$$

and $Q^* = \frac{\mu Q(1-\gamma t)c_p}{a\kappa^*}$

are the magnetic parameter, unsteadiness parameter, stretching ratio, Prandtl number, and heat source/sink parameter, respectively. The important physical quantities of this problem are skin friction coefficient and the local Nusselt number which are proportional to rate of velocity and rate of temperature respectively.

3.0 METHODOLOGY

The governing thermal boundary layer equation (3) and momentum boundary layer equation (4), with the boundary conditions (5) are transformed into a system of ordinary differential equations which are then solved numerically by using Runge-Kutta fourth-fifth order method along with shooting technique. First of all, higher order non–linear differential equations are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem applying the shooting technique. Once the problem is reduced to initial value problem, then it is solved using Runge -Kutta fourth order technique. The effects of the flow parameters on the velocity and temperature are computed, discussed and have been graphically represented in figures for various value of different parameters. Now defining new variables by the equations

$$y_1 = f, y_2 = f', y_3 = f'', y_4 = \theta, y_5 = \theta'.$$

The higher order differential equations (6), (7) and boundary conditions (8) may be transformed to five equivalent first order ordinary differential equations and boundary conditions are as follows:

$$dy_1 = y_2, dy_2 = y_3, dy_3 = \frac{y_3}{1+a^*y_4} + \frac{y_2^2}{1+a^*y_4} +$$

$$\frac{A\left(y_2 + \frac{1}{2}\eta y_3\right)}{1+a^*y_4} + \frac{M(y_2-\lambda)}{1+a^*y_4} - \frac{A\lambda}{1+a^*y_4} - \frac{\lambda^2}{1+a^*y_4},$$

$$dy_4 = y_5, dy_5 = \frac{\Pr y_1 y_5(1+A\lambda)}{1+\varepsilon y_4} - \frac{Q^* y_4}{1+\varepsilon y_4}$$

And the boundary conditions are

$$y_1 = 0, y_2 = 1, y_4 = 1, \text{ at } \eta = 0$$

$$y_2 = \lambda, y_4 = 0 \text{ as } \eta \to \infty$$

4.0 RESULTS AND DISCUSSION

For the purpose of our numerical calculation, we have chosen λ= 0.4, M = 4.5, A = 0.3, a^*= 0.4, ε =0.2, Q = 0.2 and P_r = 0.71 while the parameters are varied over range as shown in the above figures. From Fig.1 it is clearly observed that the velocity starts from maximum value at the surface and then decreases until it reaches to the minimum value at the end of the boundary layer for all the values of M. It is interesting to note that the effect of magnetic field is more prominent at the point of peak value, because the presence of M in an electrically conducting fluid introduces a force like Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as in the present problem. As a result velocity profile is decreased. Similar result arises in case of temperature profile for the increasing values of M which are shown in Fig.7. Again in Fig.2 we have observed that the negligible decreasing effect on velocity profile for increasing values of unsteadiness parameter whereas monitoring the temperature profiles of Fig. 8, it is seen that the increase of A, the temperature profile rapidly increases as a result the thermal boundary layer thickness rapidly increases. Again from Fig.4 and Fig.5 we have seen that the velocity profile is unchanged for increasing values of perturbation and heat source parameter. From the Fig.11 it is seen that the temperature profile is increased up to a certain values of eta called 'crossing over point' have completely conflicting behavior before and after that point. The value of the temperature profile for fixed values of η increases before that point and slightly decreases after that. Thus due to the increase of heat source parameter the temperature initially enhances but ultimately it increases the thickness of thermal boundary layer. Reverse result arises in case of perturbation parameter which is shown in Fig.10. The noticeable increasing effect on temperature profile for slightly increasing values of a* and λ are shown in Fig.9 and Fig.13 respectively but slightly increasing effect on velocity profile which are depicted in Fig.3 and Fig.6. Again Fig.12 clearly demonstrates that the thermal boundary layer thickness increases as the Pr increases and then decreases. Because Prandtl number Pr increase means an increase of fluid viscosity which causes a decrease in the flow velocity and as a result the temperature profile decreases.

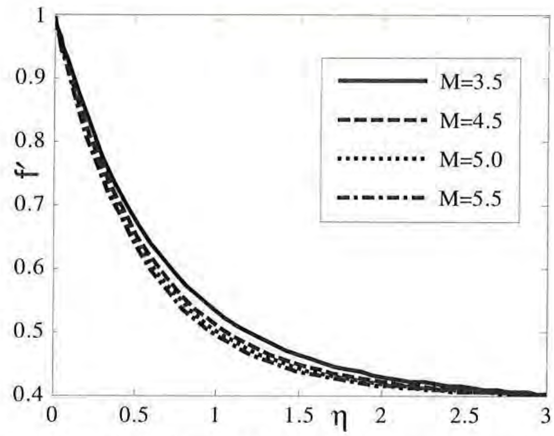

Fig. 1: Velocity profile for various values of M

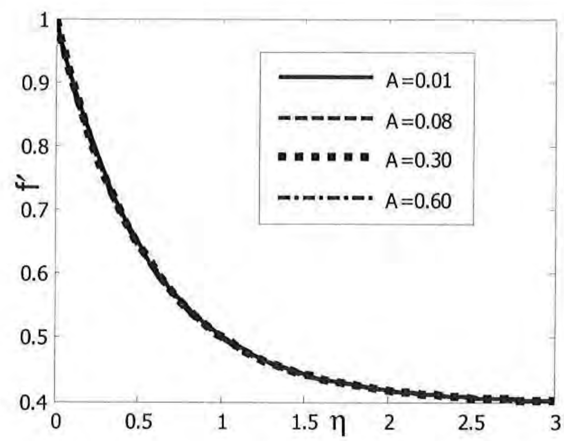

Fig. 2: Velocity profile for various values of A

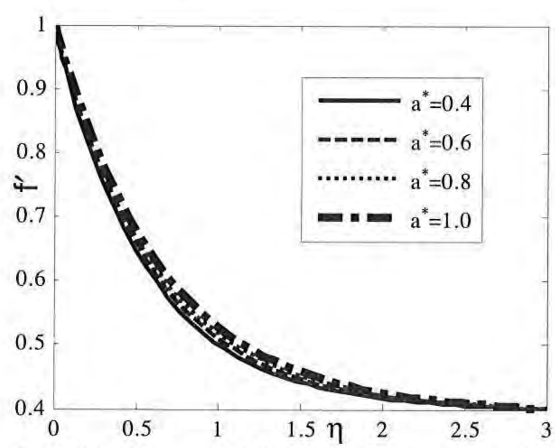

Fig. 3: Velocity profile for various values of a

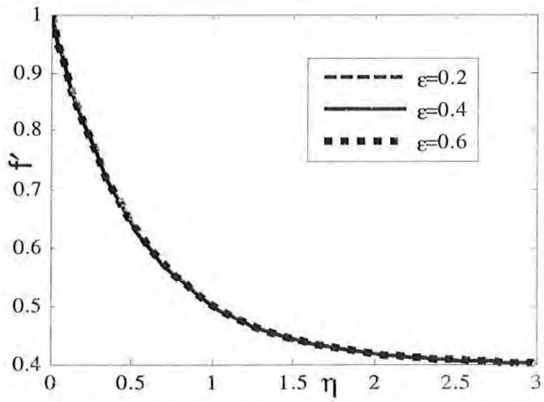

Fig. 4: Velocity profile for various values of ε

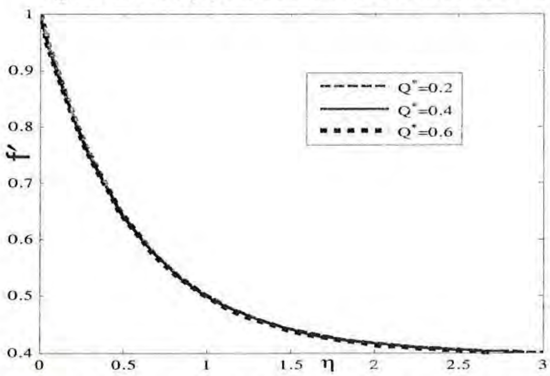

Fig. 5: Velocity profile for various values of Q^*

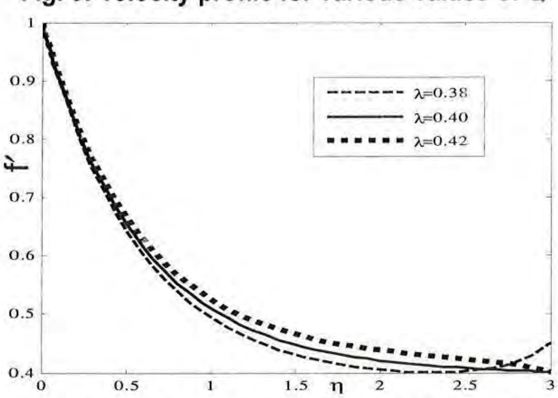

Fig. 6: Velocity profile for various values of λ

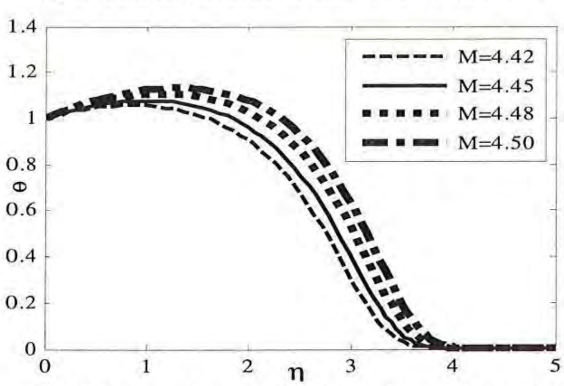

Fig. 7: Temperature profile for various values of M

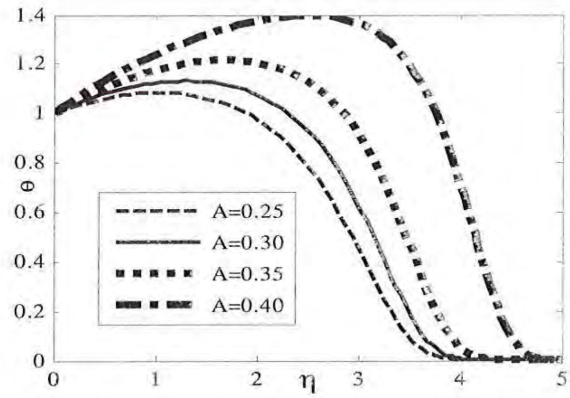

Fig. 8: Temperature profile for various values of A

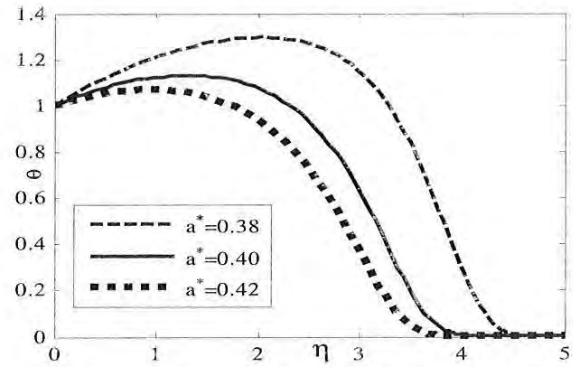

Fig. 9: Temperature profile for various values of a^*

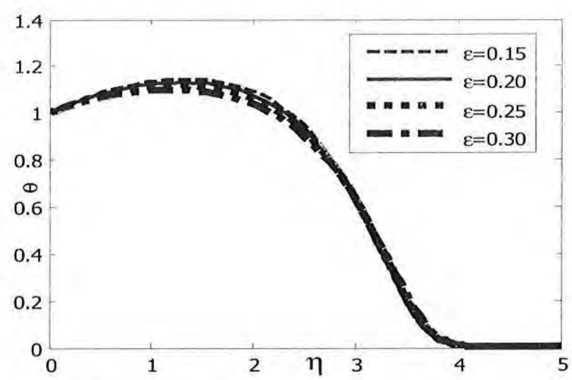

Fig. 10: Temperature profile for various values of ε

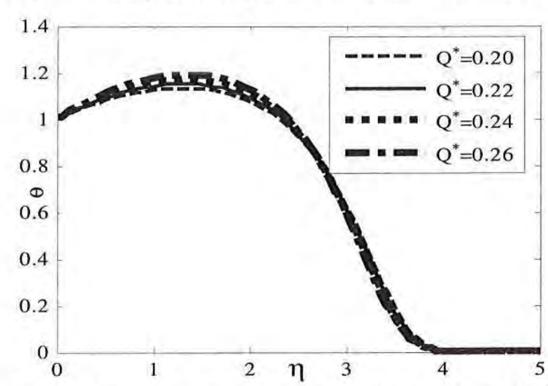

Fig. 11: Temperature profile for various values of Q^*

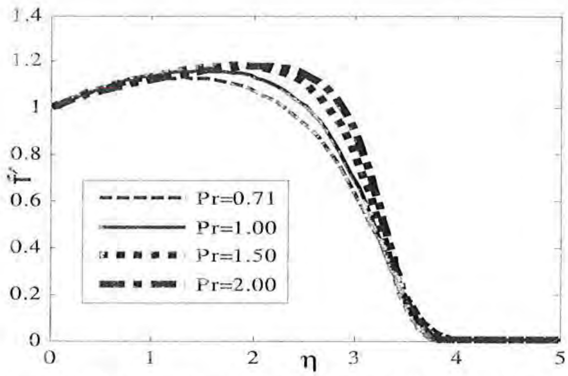

Fig. 12: Temperature profile for various values of Pr

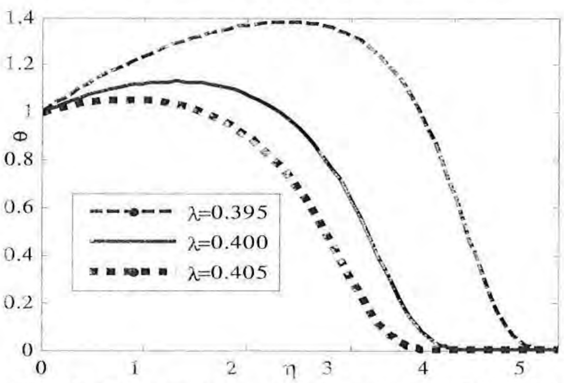

Fig. 13: Temperature profile for various values of λ

5.0 CONCLUSIONS

A numerical method has been obtained to study the flow and heat transfer in the laminar flow of an incompressible fluid over an unsteady stretching surface. The effects of various parameter like magnetic parameter M, stretching ration λ, the unsteadiness parameter A, heat source/sink parameter Q* , Prandtl number Pr and perturbation parameter ε on the heat transfer characteristics were studied. The numerical results indicated the following.

- The effect of magnetic field is more prominent at the point of peak value, because the presence of M in an electrically conducting fluid introduces a force like Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as in the present problem. As a result velocity profile is decreased for increasing values of M. Similar result arises in case of temperature profile for the increasing values of M.
- The momentum boundary layer thickness increases for the increasing values of ε and λ.
- The negligible decreasing effect on velocity profile for increasing values of unsteadiness parameter whereas the increase of A, the temperature profile rapidly increases as a result the thermal boundary layer thickness rapidly increases.
- The temperature profile is increased up to a certain values of eta called 'crossing over point' have completely conflicting behavior before and after that point. The value of the temperature profile for fixed values of η increases before that point and slightly decreases after that. Thus due to the increase of heat source parameter the temperature initially enhances but ultimately it increases the thickness of thermal boundary layer but reverse result arises in case of perturbation parameter.
- The noticeable increasing effect was observed on temperature profile for slightly increasing values of a* and λ.
- The thermal boundary layer thickness increases as the Pr increases and then decreases. Because Prandtl number Pr increase means an increase of fluid viscosity which causes a decrease in the flow velocity and as a result the temperature profile decreases.

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