CHAPTER 3

ANALYTICAL MODEL FOR BER ANALYSIS IN WDM NETWORK

3.1 Introduction

Designing an all-optical wavelength-routed WDM network requires careful consideration of the system operating conditions. Imperfections of the optical components gives rise to optical crosstalk and noise which degrade the signal quality. In this chapter the performance of a long-haul WDM network is analyzed when designed with the L-WIXC architectures presented previously considering all the important system parameters accordingly. Here, the analytical expression for Signal to Crosstalk plus Noise ratio (SCNR) considering combined effect of crosstalk and noise in the long-haul optical communication system is developed and presented for analyzing Bit Error Rate (BER) and Power Penaly (PP) performance subsequently.

3.2 Crosstalk

The narrow channel spacings in dense WDM links give rise to crosstalk, which is defined as the effect of another channel's signal on the desired channel's channel. Crosstalk can be introduced by almost any component in a WDM system, including optical filters, wavelength multiplexers and demultiplexers, optical switches, optical amplifiers and the fiber itself. It is the major limitation of all optical WDM system because it significantly degrades the system performance. The crosstalk components either have the same wavelength as the actual signal or have a different wavelength, giving rise to *Inband* crosstalk and *Interband* crosstalk. As illustrated in Fig. 3.1 for a demultiplexer function, Interband crosstalk arises when an interfering signal comes from a neighboring channel.

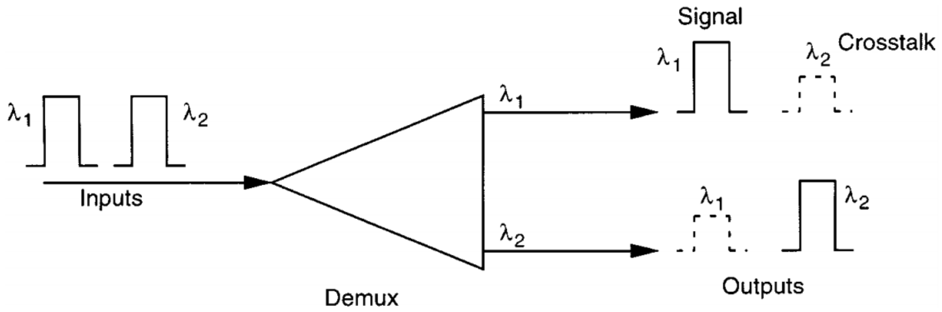

Fig 3.1 Demonstration on Interband Crosstalk

This means that the wavelength of the undesired signal is sufficiently far away from the desired signal wavelength so that the difference is greater than the electrical bandwidth of the receiver. This is not so severe because this can be removed with narrow-band filters and it produces no beating during detection.

For Inband crosstalk, the interfering signal is at the same wavelength as the desired signal. This effect is more severe than Interband crosstalk since the interference falls within the receiver bandwidth and it cannot be removed by an optical filter as it accumulates through the network. Fig. 3.2 gives an illustration of the origin of Inband crosstalk.

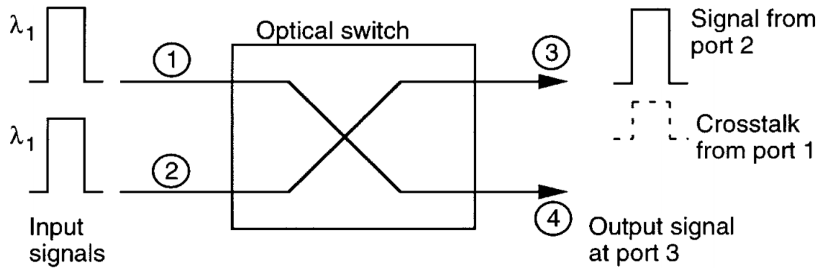

Fig 3.2 Demonstration on Inband Crosstalk

Here two independent signals, each at a wavelength λ_1, enter an optical switch. This switch routes the signal entering port 1 to output port 4, and routes the signal entering port 2 to output port 3. Within the switch, a spurious fraction of the optical power entering port 1 gets coupled to port 3, where it interferes with the signal from port 2 that gets switched there. It results in beating (fluctuation) in the signal power that falls within the receiver bandwidth and is much more damaging to the end-to-end transmission than Interband crosstalk [6]. Therefore, in this paper we focused on Inband crosstalk. The beating generated by Inband crosstalk is often modeled as a Gaussian random process [3]–[6]. This approximation is appropriate when there are a large number of crosstalk sources, by virtue of the central limit theorem. Experiment results reported in [12] show that Gaussian approximation is quite accurate for 16 or more crosstalk sources. For a signal that traverses multiple hops, the total number of crosstalk sources can easily exceed 16, which makes the Gaussian approximation a good fit. To analyze the beating effect, the worst-case approach may be used considering the fact that signals may combine coherently if they are phase correlated; when certain inband crosstalk components are phase correlated to the actual signal, they cause *coherent* crosstalk while the other crosstalk components that are not phase correlated give rise to *incoherent* crosstalk. At the same time, crosstalk components may interact with each other coherently to produce a composite crosstalk with much higher power than the (incoherent) random combination of the components [9]. The model presented in [9] is good for characterizing the coherent effect of inband crosstalk. In this thesis, an analysis of inband crosstalk in an amplified system based on the worst-case scenario and the Gaussian approximation is presented.

3.3 Assumption in Crosstalk Modeling

Crosstalk model in this analysis has been done based on the following assumption:

i) Laser phase noise has uniform distribution;

ii) Phase noise originated from different lasers are independent to each other;

iii) Digital bits are intensity modulated;

iv) Signal-spontaneous beat noise is the dominant noise effect.

v) All signal sources have the same bit rate;

vi) Integrate-and-dump filter is used at the receiver to improve the error rate;

These assumptions fit systems with optical amplifier whose spontaneous emission appears as noise. If the amplifier gain is reasonably large (>10 dB), which is normally the case for optical cross-connect systems, the receiver noise is negligible compared to the signal-spontaneous and spontaneous-spontaneous beat noise resulting from the amplifiers. The spontaneous-spontaneous beat noise can be made very small by reducing the optical bandwidth. The dominant noise is, therefore, signal-spontaneous beat noise.

The instantaneous electrical field of a signal with center frequency ω coming from port i of an OXC can be expressed as

$$\vec{E}_0(t) = Eb_i(t)cos[\omega t \ + \ \Phi i(t)]\vec{P}_i \tag{3.1}$$

Where E is the signal field amplitude that is assumed to be constant with respect to time, $b_i(t)$ is the binary data sequence with values of 0 or 1 in a bit period T, $\Phi i(t)$ is the phase noise of the laser and $\vec{P}_i$ is the unit-polarization vector of the signal. For all the architectures the amount of crosstalk power has been analyzed by choosing the desired signal at center frequency ω entering from port 1 and only the crosstalk imposed on this signal which has the same wavelength (inband crosstalk) as the same signal are considered.

3.4 Crosstalk Analysis of Share Per Node L-WIXC

In this section, crosstalk contributions in L-WIXC architecture that is presented in Fig 2.2 (Share per Node Architecture) are identified so that it can be used for all subsequent analysis of crosstalk contributions in other architectures of L-WIXCs. However, their values depend on the coherence relations of the various components and these relations may change with the relative delays of the components.

3.4.1 Coherence Property

The laser output, which is the result of electron state transition from one energy band to another is emitted at a specific wavelength corresponding to a resonance mode in the laser's cavity. But this wavelength will actually fluctuate around a center wavelength because electrons can leave and enter slightly different energy levels within the same band. The amount of fluctuation in the wavelength is called linewidth and can be accounted for by the laser phase noise. The length of time that coherence is maintained is called the coherence time. The length that the signal could travel in a vacuum during that time is called the coherence length. In statistical terms, a random function becomes incoherent when its autocorrelation vanishes. Statistically, the coherence time and linewidth are related by,

$$\text{coherence time} = \frac{\lambda^2}{C\Delta\lambda}$$

where λ is the center wavelength, $\Delta\lambda$ is the linewidth and C is the speed of light in a vacuum. A simple laser may have a coherence time of 0.5 ns and a coherence length of 15 cm. In an OXC, the delay τ experienced by the various components depend on the OXC hardware. Again, for a high-speed communication system, the bit duration T may be less than the laser coherence time t_1, e.g. $T = 0.4$ ns for 2.5 Gbit/s transmission.

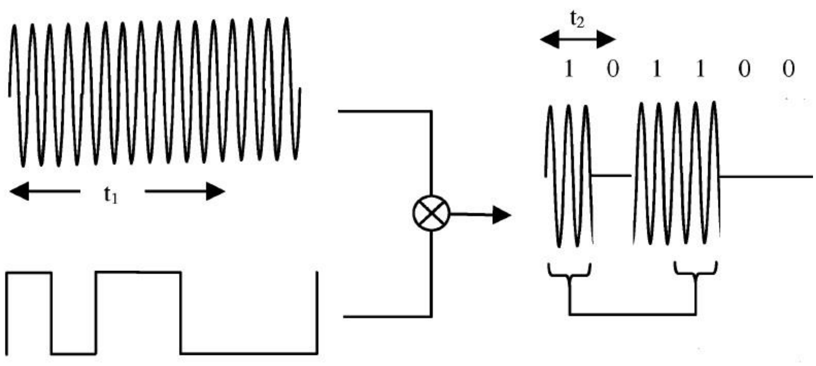

Fig 3.3 Coherence time of Laser (t_1) and Modulated Light (t_2)

Considering the continuous wave (CW) light and bit stream in Fig. 3.3, although the modulated light is no longer coherent beyond t_2 (the modulated light's coherence time), the portions of CW within the first and fourth bits, for instance, remain correlated if both are bit "1" and apart from each other less than t_1 (laser coherence time). To account for the various possibilities, the crosstalk in an OXC is studied for following two different cases depending on the relationship between the relative propagation delay and the laser's phase coherence time in the subsequent analysis:

Case 1: $\tau < t_1$ and $\tau << T$, Bit period or Coherent case

Case 2: $\tau < t_1$ and $\tau > T$, Bit period or Incoherent case

Such crosstalk will be generated when the optical propagation delay differences of optical paths in an OXC do not exceed the coherent time of the lasers. While causing fluctuation of signal power, coherent crosstalk may cause noise or not, depending on the relationship between the optical propagation delay differences and the time duration of one bit of the signal. Incoherent crosstalk may cause very high noise power, because it can be a coherent combination of crosstalk contributions.

3.4.2 Crosstalk in Share Per Node Architecture

This architecture consists of star couplers, tunable Filters and space division switches. The crosstalk mechanism in different optical components of this architecture is explained bellow.

a. Tunable Filter

The actual signal in question is combined with M-1 signals at different wavelengths by a star coupler at the output of OXC as depicted in Fig. 2.2. These signals carry along crosstalk components having the same wavelength as the actual signal with them, which can be traced back to the tunable filters. Due to imperfect filtering, $M-1$ crosstalk components at ω leaked through the filters and mix with the actual signal at the star coupler after passing through the switches, as shown in Fig. 2.2. Let X_i $(i = 1,2, \ldots, N)$ be the number of crosstalk components at ω that are leakages of the signal entering the OXC at input port i. Then, each X_i is an integer satisfying,

$$\sum_{i=1}^{N} X_i = M - 1, \quad 1 \le X_i \le M - 1 \tag{3.2}$$

b. Space-Division Switch

In a practical optical switch, each signal produces leakage to unintended output inducing crosstalk to the actual signal. As per fig 2.2, there are $N-1$ inband crosstalk components at ω leaking from the switches. These are contributed by signals entering from different input ports. In a wavelength converter for a signal converted to ω, there must be another signal originally at ω being converted to another wavelength. Therefore, in the worst case, assuming C converters, there are K crosstalk components, with $K = min(N-1, \lfloor C\ /\ 2 \rfloor)$ which leak from the second stage switches. Assuming the converted signal is free from the crosstalk carried with it before wavelength conversion. Assuming that the OXC is fully loaded, in the worst case, the actual signal will be interfered by $M-1$ crosstalk components leaking from the tunable filters and another $N-1$ crosstalk components leaking from the switches. These components traverse the OXC via different paths and, thus, have different propagation delays. Assuming intensity modulation, the electric field, which includes the influence of the crosstalk, is given by

$$\vec{E}(t) = Eb_1(t) cos[\omega t + \Phi_1(t)]\vec{P}_1 + \sum_{i=1}^{N}\sum_{j=1}^{X_i} \sqrt{\delta}\, Eb_i(t-\tau_{ij})\cos[\omega(t-\tau_{ij}) + \varphi_i(t-\tau_{ij})]\vec{P}_{ij} + \sum_{i=2}^{N}\sqrt{\varepsilon}\; Eb_i(t-\tau_{ix})\cos[\omega(t-\tau_{ix}) + \varphi_i(t-\tau_{ix})]\overrightarrow{P_{ix}} + \sum_{i=2}^{K}\sqrt{\varepsilon'}\; Eb_i'(t-\tau_i')\cos[\omega(t-\tau_i') + \varphi_i'(t-\tau_i')]\overrightarrow{P_i'} \tag{3.3}$$

where,

τ_{ij} Propagation delay relative to the actual signal of the j th crosstalk component leaking from the signal entering from port of the OXC at the tunable filter,

$\vec{P}_{ij}$ The unit polarization vector of crosstalk component τ_{ij} ,

τ_{ix} Propagation delay relative to the actual signal of the crosstalk component leaking from the signal entering from port of the OXC at the optical switch,

$\vec{P}_{ix}$ The unit polarization vector of crosstalk component τ_{ix},

δ Optical power relative to the actual signal for the crosstalk components leaked at a tunable filter,

ε Optical power relative to the actual signal for the crosstalk components leaked at an optical switch,

ε' Optical power relative to the actual signal for the crosstalk components leaked at second stage switch.

Equation (3.3) can be rewritten as,

$$\vec{E}(t) = Eb_1(t)cos[\omega t \ + \Phi_1 \ (t)]\vec{P}_1 + E\sum_{j=1}^{X_i}\sqrt{\delta}\, b_1 t - \tau_{1j)} \cos[\omega(t-\tau_{1j)} + \varphi_1\big(t-\tau_{1j)}\big]\vec{P}_{1j}$$

$$+ \sum_{i=2}^{N} E\{\sum_{j=1}^{X_i}\sqrt{\delta}\, b_i(t-\tau_{ij)} \cos[\omega(t-\tau_{ij)} + \varphi_i\big(t-\tau_{ij)}\big]\vec{P}_{ij} + \sqrt{\varepsilon}\, b_i(t$$

$$-\tau_{ix}) \cos[\omega(t-\tau_{ix)} + \varphi_i\big(t-\tau_{ix)}\big]\vec{P}_{ix}\} + E\sum_{i=2}^{K}\sqrt{\varepsilon'}\ b_i'(t$$

$$-\tau_i') \cos[\omega(t-\tau_i') + \varphi_i'\big(t-\tau_i'\big]\overrightarrow{P_i'} \qquad (3.4)$$

The first term in equation (3.4) is the actual signal while the second and third terms correspond to the crosstalk contributed by leakages from the filters; the fourth term corresponds to crosstalk by first stage switches and fifth term for second stage switches respectively. In the second term, $\sum_{j=1}^{X_i}\sqrt{\delta}\, Eb_i(t-\tau_{ij)} \cos[\omega(t-\tau_{ij)} + \varphi_i\big(t-\tau_{ij)}\big]\vec{P}_{ij}$ is the leakage contributed by the signal from input link i. These components may mix coherently with each other, since they have originated from the same signal and their phases are correlated. Considering an intensity modulation/direct detection

(IM/DD) system; when the signal is received by a photodetector, the induced photocurrent $I(t)$ is proportional to $\left|\vec{E}(t)\right|^2$. If the receiver uses an integrate-and-dump filter, the decision variable is given by

$$Y = \int_{nT}^{(n+1)T} I(t)dt$$

where the limits of the integration are aligned with the bits. All noise processes are assumed to be Gaussian. Let Y_0 be the decision variable when there is no crosstalk and $J = \frac{Y}{E(Y_0)}$, where $E(.)$ gives the mean of a random variable.

3.4.3. Crosstalk Analysis

Some of the crosstalk components originate from the same signal and their phases are correlated when their relative delays are less than the laser's coherence time. They mix coherently with each other to form a composite crosstalk and this results in a more prominent crosstalk effect than the incoherent combination of the components. In this case, the time-delay terms cannot be neglected, but the terms $\varphi(t-\tau)$ can be approximated by $\varphi(t)$. Equation (3.4) can then be written as,

$$\vec{E}(t) = Eb_1(t)cos[\omega t + \Phi_1(t)]\vec{P}_1 + E\sum_{i=1}^{X_1}\sqrt{\delta}\, E_{1i}(t-\tau_{1j)} \cos[\omega(t-\tau_{i1)} + \varphi_1(t]\vec{P}_{1j} + \sum_{i=2}^{N} E\left\{\sum_{j=1}^{X_i}\sqrt{\delta}\, b_i(t-\tau_{ij)} \cos[\omega(t-\tau_{ij)} + \varphi_1(t]\vec{P}_{ij} + \sqrt{\varepsilon} b_i(t - \tau_{ix)} \cos[\omega(t-\tau_{ix)} + \varphi_i(t)]\overrightarrow{P_{ix}}\right\} \quad (3.5)$$

The second term represents the composite crosstalk that consists of the components leaking from the actual signal and is, therefore, coherent with the actual signal. The other composite crosstalk contributions are incoherent, each of which is a coherent combination of a number of crosstalk leaking from the same signal.

$$J_2 \approx \frac{1}{T}\int_{nT}^{(n+1)T} b_1^2(t)dt + \frac{2}{T}\sum_{j=1}^{X_1}\sqrt{\delta}\int_{nT}^{(n+1)T} b_1(t)\, b_1\big(t-\tau_{1j}\big)dt\ \cos D_{1j}\cos\vartheta_{1j} - \varphi_{id}(t)]$$
$$+\frac{2}{T}\sum_{i=2}^{N}\left\{\sum_{j=1}^{X_i}\sqrt{\delta}\int_{nT}^{(n+1)T} b_1(t)\, b_i\big(t-\tau_{ij}\big)dt\cos\big[\varphi_1(t)-\varphi_i(t)+D_{ij}\big]\cos\vartheta_{ij}\right.$$
$$\left.+\sqrt{\varepsilon}\int_{nT}^{(n+1)T} b_1(t)\, b_i(t-\tau_{ix})dt\cos[\varphi_1(t)-\varphi_i(t)+D_{ix}]\cos\vartheta_{ix}\right\}$$
$$+\ \frac{2}{T}\sum_{i=2}^{K}\sqrt{\varepsilon'}\int_{nT}^{(n+1)T} b_1(t)\, b_1(t-\tau_i)dt\cos\big[\varphi_1(t)-\varphi'_i(t)+D'_i\big]\cos\vartheta'_i \quad (3.6)$$

The decision variable J_2 is given by (3.6) where D_{ij}=$\omega\tau_{ij}$, D_{ix}=$\omega\tau_{ix}$, $D'_i = \omega\tau'_i$, $\cos\vartheta_{ij} = \overrightarrow{P_1}.\overrightarrow{P_{ij}}$, $\cos\vartheta_{ij} = \overrightarrow{P_1}.\overrightarrow{P_{ix}}$ and $\cos\vartheta'_i = \vec{P}_1.\vec{P}'_i$. The case of interest is when $b_1(t) = 1$. The crosstalk power is closely related to the relative propagation delays experienced by individual components. The results for two different scenarios of interest are summarized next. The details can be found in Appendix A.

a. **Coherent Case:$\boldsymbol{\tau \ll T}$** : In this special case in which the relative delays are negligible, the bit patterns of the components from the same source are almost identical. All $b(t-\tau)$ are approximately equal to $b(t)$. This is the case when the components are delayed by almost the same amount and they interfere with each other constructively to produce high power. In this case the mean and variance of J may be expressed as

$$E(J_1) = 1 + 2\sqrt{\delta}\sum_{j=1}^{X_1}\cos D_{1j}\cos\vartheta_{1j} \quad (3.7)$$

$$\sigma_1^2 = \frac{2}{3}\sum_{i=2}^{N}\left(\sqrt{\delta}\sum_{j=1}^{X_i}\cos D_{ij}\cos\vartheta_{ij} + \sqrt{\varepsilon}\sum_{j=1}^{X_i}\cos D_{ix}\cos\vartheta_{ix}\right)^2$$
$$+\ \frac{2}{3}\sum_{i=2}^{N}\left(\sqrt{\delta}\sum_{j=1}^{X_i}\sin D_{ij}\cos\vartheta_{ij} + \sqrt{\varepsilon}\sum_{j=1}^{X_i}\sin D_{ix}\cos\vartheta_{ix}\right)^2$$
$$+\ \frac{2}{3}\sum_{i=1}^{K}\varepsilon'\cos^2\vartheta'_i \quad (3.8)$$

The crosstalk incurs maximum PP when all $\cos\vartheta$ terms are equal to 1 or -1 and $X_1 = 0$, which gives $E(J) = 1$. It can be shown that

$$max\sum_{i=1}^{n}(Ax_i + B)^2 = (AL_n + B)^2 + (n-1)B^2, [\sum_{i=1}^{n} x_i = L_n] \tag{3.9}$$

$$max\left(\sum_{j=1}^{n} A_j \cos\theta_j \right)^2 + \left(\sum_{j=1}^{n} A_j \sin\theta_j \right)^2 = \left(\sum_{j=1}^{n} A_j\right)^2 \tag{3.10}$$

Applying this, we have

$$max(\sigma_1^2) = max\left[\frac{2}{3}\{\sum_{i=2}^{N}\left(X_i\sqrt{\delta} + \sqrt{\varepsilon}\right)^2 + K\varepsilon'\}\right]$$

$$= \frac{2}{3}\left[(M-1)\sqrt{\delta} + \sqrt{\varepsilon}\right]^2 + \frac{2}{3}(N-2)\varepsilon + \frac{2}{3}k\varepsilon' \tag{3.11}$$

b. **Incoherent Case: $\boldsymbol{\tau > T}$** : The delays experienced by any two components differ from each other by more than a one-bit period. All $b(t-\tau)$ become uncorrelated with each other and with the actual signal $b_1(t)$, the mean and variance of J can be shown to be,

$$E(J_2) = 1 + \sqrt{\delta}\sum_{j=1}^{X_1}\cos D_{1j}\cos\vartheta_{1j} \tag{3.12}$$

$$\sigma_2^2 = \frac{1}{3}\delta\left(\sum_{j=1}^{X_1}\cos^2 D_{1j}\cos^2\vartheta_{1j}\right)\frac{1}{2}\sum_{i=2}^{N}\left(\sqrt{\delta}\sum_{j=1}^{X_i}\cos D_{ij}\cos\vartheta_{ij} + \sqrt{\varepsilon}\sum_{j=1}^{X_i}\cos D_{ix}\cos\vartheta_{ix}\right)^2 + \frac{1}{2}\sum_{i=2}^{N}\left(\sqrt{\delta}\sum_{j=1}^{X_i}\sin D_{ij}\cos\vartheta_{ij} + \sqrt{\varepsilon}\sum_{j=1}^{X_i}\sin D_{ix}\cos\vartheta_{ix}\right)^2 + \frac{1}{6}\sum_{i=2}^{N}\left(\delta\sum_{j=1}^{X_i}\cos^2\vartheta_{ij} + \varepsilon\cos^2\vartheta_{ix}\right) + \frac{2}{3}\sum_{i=1}^{K}\varepsilon'\cos^2\vartheta_i' \tag{3.13}$$

When all $\cos\vartheta$ terms are equal to -1 or 1 and $X_1 = 0$, the PP is at its maximum. Applying this in (3.9) and (3.10), we get

$$max(\sigma_2^2) = max\left[\frac{1}{2}\sum_{i=2}^{N}\left(X_i\sqrt{\delta} + \sqrt{\varepsilon}\right)^2 + \frac{1}{6}\sum_{i=2}^{N}(X_i\,\delta + \varepsilon) + \frac{2}{3}K\varepsilon'\right]$$

$$= \frac{1}{2}\left[(M-1)\sqrt{\delta} + \sqrt{\varepsilon}\right]^2 + \frac{1}{2}(N-2)\varepsilon + \frac{1}{6}(M-1)\delta + \frac{1}{6}(N-1)\varepsilon + \frac{2}{3}k\varepsilon'$$

$$= \frac{1}{2}\left[(M-1)\sqrt{\delta} + \sqrt{\varepsilon}\right]^2 + \frac{1}{6}(4N-7)\varepsilon + \frac{1}{6}(M-1)\delta + \frac{2}{3}k\varepsilon' \qquad (3.14)$$

3.5 Analysis of Crosstalk for Other L-WIXC Architectures

Using the framework described in Section 3.4, the crosstalk analysis of other L-WIXC architectures is done in this section by identifying the inband crosstalk sources.

3.5.1 Share Per Link Architecture

Compared to Share per Node Architecture depicted in previous section, in this architecture there are dedicated set of converters for each link. The signals that need wavelength conversion have to go through wavelength converter before they pass through space division switches. At the tunable filters, $M-1$ inband crosstalk components mix with the actual signal at the star coupler after passing through the switches. In a real SDS, each crosstalk causes unintended outputs, including crosstalk to the actual signal. In this case there are $N-1$ inband crosstalk components at ω leaking from the switches. These are contributed by signal entering from different input ports. If the OXC is fully loaded, in the worst case, the actual signal will be interfered by $M - C_n$ crosstalk components leaking from the tunable filters and another $N-1$ crosstalk components leaking from the switches. The components traverse the OXC via different paths and thus, have different propagation delays. Therefore, applying the method of analysis described in Section 3.4, crosstalk generated in Share per Link architecture can be written as,

a) Coherent Case

$$\max(\sigma_1^2) = \frac{1}{2}\left\{(\mathrm{M} - \mathrm{C_n})\sqrt{\delta} + \sqrt{\varepsilon}\right\}^2 + \frac{2}{3}(\mathrm{N} - 2)\varepsilon \tag{3.15}$$

b) Incoherent Case

$$\max(\sigma_2^2) = \frac{1}{2}\left\{(\mathrm{M} - \mathrm{C_n})\sqrt{\delta} + \sqrt{\varepsilon}\right\}^2 + (4\mathrm{N} - 7)\frac{\varepsilon}{6} + (\mathrm{M} - \mathrm{C_n})\frac{\delta}{6} \tag{3.16}$$

3.5.2 DCS-1 Architecture

This architecture is same as Share per Node architecture except that instead of Space Switch here, Delivery and coupling Switch (DCS) is used. The actual signal in question is combined with $M - 1$ signals at different wavelengths by a star coupler at the output. These signals carry with them crosstalk components having the same wavelength as the actual signal, which can be traced back to the tunable filters. Due to imperfect filtering, $M - 1$ crosstalk components at ω leaked through the filters and mix with the actual signal at the star coupler after passing through the switches. In a real SDS, each crosstalk will leak to the unattended outputs, including crosstalk to the actual signal. There are $N - 1$ inband crosstalk components at ω leaking from the first stage switches. These are contributed by signal entering from different input ports. In the wavelength converter for a signal converted to ω, there must be another signal originally at ω being converted to another wavelength. Therefore, in the worst case, assuming C converters, there are K crosstalk components, with $K = \min\left(\mathrm{N} - 1, \left\lfloor \frac{C}{2} \right\rfloor\right)$ which leak from the second stage switches. The converted signal is free from the crosstalk carried with it before wavelength conversion. Assuming that the OXC is fully loaded, in the worst case, the actual signal will be interfered by $M - 1$ crosstalk components leaking from the tunable filters, another $N - 1$ crosstalk components leaking from the switches and L components leaking from the second stage switches. The components traverse the OXC via different paths and, thus, have different propagation delays. Therefore, applying the method of analysis described in Section 3.4, crosstalk generated in DCS-1 architecture can be written as,

a) Coherent Case

$$\max(\sigma_1^2) = \frac{2}{3}[\{(M-1)\sqrt{\delta} + \sqrt{\varepsilon}\}^2 + (N-2)\varepsilon + k\varepsilon'] \qquad (3.17)$$

b) Incoherent Case

$$\max(\sigma_2^2) = \frac{1}{2}\left\{(M-1)\sqrt{\delta} + \sqrt{\varepsilon}\right\}^2 + (4N-7)\frac{\varepsilon}{6} + (M-1)\frac{\delta}{6} + \frac{2}{3}k\varepsilon' \qquad (3.18)$$

3.5.3 DCS-2 Architecture

This architecture is same as Share per Link architecture except that instead of Space Switch here, Delivery and coupling Switch (DCS) is used. In this architecture there are dedicated set of converters for each link and before DCS switches the signals that need wavelength conversion have to go through the wavelength converters. Like previous architecture, in this architecture also the actual signal is also combined with $M-1$ signals at different wavelengths by a star coupler at the output. These signals also carry with them inband crosstalk components, which can be traced back to the tunable filters. Due to imperfect filtering, $M-1$ crosstalk components at ω leaked through the filters. The signals which go through the conversion process are free from crosstalk. So there is $M - V_n$ inband crosstalk components that actually mix with the actual signal at the star coupler after passing through the switches. In a real SDS, each crosstalk causes unintended outputs, including crosstalk to the actual signal. In this case there are also $N-1$ inband crosstalk components at ω leaking from the switches. These are contributed by signal entering from different input ports. In this case we also assume that the OXC is fully loaded, in the worst case, the actual signal will be interfered by $M - C_n$ crosstalk components leaking from the tunable filters and another $N - 1$ crosstalk components leaking from the switches. The components traverse the OXC via different paths and, thus, have different propagation delays. Therefore, applying the method of analysis described in Section 3.4, crosstalk generated in DCS-2 architecture can be written as,

a) Coherent Case

$$\max(\sigma_1^2) = \frac{2}{3}\left\{(\mathrm{M} - \mathrm{C_n})\sqrt{\delta} + \sqrt{\varepsilon}\right\}^2 + \frac{2}{3}(\mathrm{N} - 2)\varepsilon \quad (3.19)$$

b) Incoherent Case

$$\max(\sigma_2^2) = \frac{1}{2}\left\{(\mathrm{M} - \mathrm{C_n})\sqrt{\delta} + \sqrt{\varepsilon}\right\}^2 + (4\mathrm{N} - 7)\frac{\varepsilon}{6} + (\mathrm{M} - C_\mathrm{n})\frac{\delta}{6} \quad (3.20)$$

3.5.4 Wavelength Switch-based Architecture

In this architecture the signals have to go through two stage switching structure before combined at the output star coupler. In first stage Space-Division Switch there are $N - 1$ inband crosstalk components due to leakage to the unattended output which include crosstalk to the actual signal. These are contributed by signal entering from different input ports, The actual signal is also combined with $M - 1$ signals at different wavelengths by a star coupler at the output. These signals also carry inband crosstalk components with them, due to imperfect filtering. $M - 1$ crosstalk components at ω leaked through the filters. The signals which go through the conversion process are free from crosstalk. So there is $M - C_n$ inband crosstalk components that actually mix with the actual signal at the star coupler after passing through the second stage switches. In the second stage Space-Division Switch there are also $N - 1$ leakages due to presence of other signals at wavelength ω due to imperfect isolation. These crosstalk components also mix with the signal in question at the output of the OXC. In this case we also assume that the OXC is fully loaded, in the worst case, the actual signal will be interfered by $M - C_n$ crosstalk components leaking from the tunable filters and $N - 1$ crosstalk components leaking from the switches each. The components traverse the OXC via different paths and, thus, have different propagation delays.

a) Coherent Case

$$\max(\sigma_1^2) = \frac{2}{3}\{(M - C_n)\sqrt{\delta} + 2\sqrt{\varepsilon}\}^2 + \frac{4}{3}(N - 2)\varepsilon \quad (3.21)$$

b) Incoherent Case

$$\max(\sigma_2^2) = \frac{1}{2}\{(M - 1)\sqrt{\delta} + \sqrt{\varepsilon}\}^2 + \frac{1}{3}(4N - 7)\varepsilon + \frac{1}{6}(M - C_n)\delta \quad (3.22)$$

3.5.5 MWSF-based Architecture

This architecture composed of multi-wavelength selective filter (MWSF), tunable filters & star couplers. The MWSFs will be configured such that signals of same wavelength are never led to the same coupler. There are N identical intermediate modules. In each of the modules only C_n tunable filters are followed by wavelength converters. There are leakages in the MWSF & the tunable filter. As in each of the modules C_n tunable filters are followed by wavelength converters then the number of crosstalk component at the output equal to $M - C_n$ which leak from the same signal. There are also $N - 1$ crosstalk components that leak from different input ports having same wavelength as the main signal. Therefore, applying the method of analysis described in Section 3.4, crosstalk generated in MWSF-based architecture can be written as,

a) Coherent Case

$$\max(\sigma_1^2) = \frac{2}{3}\delta\{(M - C_n + 1)^2 + (2N - 3)\} \quad (3.23)$$

b) Incoherent Case

$$\max(\sigma_2^2) = \frac{1}{2}\delta(M - C_n + 1)^2 + \frac{2}{3}(2N - 3)\delta + \frac{1}{6}(M - C_n + 1)\delta \quad (3.24)$$

3.6 Noise Generated in a Multi-hop WDM Network

In a wavelength routed WDM network, while a signal travels multiple node through the network, generally it suffers from various losses before reaching the target node. Therefore, optical amplifiers are often used to overcome losses in a long-haul optical communication system where a number of add/drop occurs in multiple nodes. The impact of optical amplifier used in WDM network has been discussed in details in [7]. Assuming intensity modulation/direct detection (IM/DD) system and considering both receiver noise and beat noise have Gaussian probability function, the variances of thermal noise, shot noise, signal-spontaneous beat noise and spontaneous-spontaneous beat noise power can be expressed as [8],

$$\sigma_{th}^2 = \frac{4kTf_nB_e}{r_L} \tag{3.25}$$

$$\sigma_S^2 = 2eB_e[r_DGP_{in} + i_d + i_b + r_Dn_{sp}hf(G-1)B_o] \tag{3.26}$$

$$\sigma_{s-sp}^2 = 4r_D^2GP_{in}n_{sp}hf_c(G-1)B_e \tag{3.27}$$

$$\sigma_{sp-sp}^2 = 2r_D^2\left[n_{sp}hf_c(G-1)\right]^2(2B_e - B_o)B_e \tag{3.28}$$

where σ_{th}^2 represents thermal noise power, σ_S^2 is shot noise power, σ_{s-sp}^2 is the signal-ASE beat noise power, and σ_{sp-sp}^2 is the ASE-ASE beat noise power of the receiver, respectively. The spontaneous-spontaneous beat noise can be made insignificant by making the optical bandwidth B_0 equal to twice the electrical bandwidth B_e. This can be done by filtering the amplifier noise before it reaches the receiver. So the dominant noise component is usually signal-ASE beat noise.

3.7 BER Performance Analysis

We now analyse the combined effect of crosstalk and noise in the long-haul optical communication system considering the L-WIXC structures depicted in Chapter 2. We have investigated the combined trend of noise and crosstalk that deteriorate the signal and developed an analytical expression for the Signal to Crosstalk plus Noise ratio (SCNR) and BER considering both the coherent and incoherent cases of L-WIXC architectures in a multi-hop WDM network based on the discussion presented in previous sections. To evaluate the performance, we have redefined the equation of total crosstalk plus noise power of a multi-hop WDM network with OXC structure as such,

$$\sigma_T = \sqrt{N_h\sigma_x^2 + \sigma_{Th}^2 + \sigma_{Sh}^2 + N_h\sigma_{sigsp}^2 + N_h\sigma_{spsp}^2} \tag{3.29}$$

where N_h represents the number of hops travelled by the signal. The photocurrent ($I_s = Gr_DP_s$) is proportional to the pre-amplified optical power. Therefore, the expression for SCNR and BER of a multi-hop network can be written as,

$$SCNR = I_s^2/\sigma_T^2 \tag{3.30}$$

$$BER = \frac{1}{2}erfc(\frac{1}{2\sqrt{2}}\sqrt{SCNR}) \tag{3.31}$$

where G is the preamplifier gain, r_D is the responsivity, P_s is optical power of the main signal, and σ_T is the total noise power of the receiver in optically amplified system.

3.8 Power Penalty

The impact of crosstalk in a wavelength division multiplexing (WDM) system may be quantified by considering the power penalty (PP). The PP is defined as the additional power (In decibels) needed for the signal to achieve the same bit error rate as that without crosstalk and noise. That is,

$$\mathbf{PP} = \mathbf{P}_{\text{rec(with crosstalk plus noise)}} - \mathbf{P}_{\text{rec(without crosstalk and noise)}} \tag{3.32}$$

The distance a signal can travel maintaining a specific BER level induced due to components of the network and noise induced in an optically amplified system is determined based on sustainable PP. In the next chapter, the BER and Power Penalty (PP) performance is presented and analyzed.

3.9 Summary

In this chapter, an analytical approach is presented to evaluate the combined effect of crosstalk and noise that deteriorate the signal and expression for the SCNR and BER in a multi-hop WDM network is developed. The crosstalk in six different types of L-WIXC architectures considering both the coherent and incoherent cases of L-WIXC architectures in a multi-hop WDM network is investigated based on the discussion presented in previous chapter. The equation of total crosstalk plus noise power of a multi-hop WDM network with L-WIXC architecture is redefined. The expression for BER performance and Power Penalty performance is presented based on the redefined equation in the next chapter. It is important to determine how much a signal can travel maintaining a specific BER level considering the combined effect of crosstalk induced due to components of the network and noise induced in an optically amplified system.